Heat and Mass Transfer in MHD Visco-Elastic Fluid Flow through a Porous Medium over a Stretching Sheet with Chemical Reaction

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Abstract

This paper presents the study of convective heat and mass transfer characteristics of an incompressible MHD visco-elastic fluid flow immersed in a porous medium over a stretching sheet with chemical reaction and thermal stratification effects. The resultant governing boundary layer equations are highly non-linear and coupled form of partial differential equations, and they have been solved by using fourth order Runge-Kutta integration scheme with Newton Raphson shooting method. Numerical computations are carried out for the non-dimensional physical parameters. Here a numerical has been carried out to study the effect of different physical parameters such as visco-elasticity, permeability of the porous medium, magnetic field, Grashof number, Schmidt number, heat source parameter and chemical reaction parameter on the flow, heat and mass transfer characteristics.

1. Introduction

In recent years, a great deal of interest has been generated in the area of heat and mass transfer of the boundary layer flow over a stretching sheet, in view of its numerous and wide-ranging applications in various fields like polymer processing industry in particular in manufacturing process of artificial film and artificial fibers and in some applications of dilute polymer solution. Sakiadis [1,2] was the first study of boundary layer problem assuming velocity of a boundary sheet as constant. This work is followed by the pioneering work of Tsou *et al*. [3] studied the flow and heat transfer developed by continuously moving surface both analytically and experimentally, in which the flow is caused by an elastic sheet moving in its own plane with a velocity varying linearly with the distance from a fixed point studied by Crane [4]. Chakrabarti and Gupta [5] studied the temperature distribution in this MHD boundary layer flow over a stretching sheet in the presence of suction. There are several

extensions to this problem, which include consideration of more general stretching velocity and the study of heat transfer [6-14].

In view of increasing importance of non-Newtonian flows, a great deal of work has been carried out to find the similarity solution of viscoelastic fluid flow over impervious stretching boundary. Rajagopal *et al*. [15] examined for a special class of visco-elastic fluids known as second order fluids. Siddappa *et al*. [16] studied the flow of visoelastic fluids of the type Walter's liquid B past a stretching sheet. Abel and Veena [17] studied the viscoelasticity on the flow and heat transfer in a porous medium over a stretching sheet. All these studies deals with the studies concerning non-Newtonian flows and heat transfer in the absence of magnetic fields, but present years we find several industrial applications such as polymer technology and metallurgy [18], where the magnetic field is applied in the visco-elastic fluid flow. Sarpakaya [19] was mostly first researcher to investigate MHD flows of non-Newtonian fluids, Andersson [20] investigated the flow problem of electrically conducting viscoelastic fluid past a flat and impermeable elastic

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sheet and later his work is extended by many authors [21-25].

Chemical reactions usually accompany a large amount of exothermic and endothermic reactions. These characteristics can be easily seen in a lot of industrial processes. Recently, it has been realized that it is not always permissible to neglect the convection effects in porous constructed chemical reactors [26]. The reaction produced in a porous medium was extraordinarily in common, such as the topic of PEM fuel cells modules and the polluted underground water because of discharging the toxic substance, etc.

Fourier's law, for instance, described the relation between energy flux and temperature gradient. In other aspects, Fick's law was determined by the correlation of mass flux and concentration gradient. Moreover, it was found that energy flux can also be generated by composition gradients, pressure gradients, or body forces. The energy flux caused by a composition gradient was discovered in 1873 by Dufour and was correspondingly referred to the Dufour effect. It was also called the diffusion-thermo effect. On the other hand, mass flux can also be created by a temperature gradient, as was established by Soret. This is the thermal-diffusion effect. In general, the thermal-diffusion and the diffusion-thermo effects were of a smaller order of magnitude than the effects described by Fourier's or Fick's law and were often neglected in heat and mass transfer processes. There were still some exceptional conditions. The thermal-diffusion effect has been utilized for isotope separation and in mixtures between gases with very light molecular weight (H_2, He) and of medium molecular weight (N_2, air), the diffusion-thermo effect was found to be of a magnitude such that it may not be neglected in certain conditions [27]. In recent years, Kandasamy et al. studied the heat and mass transfer under a chemical reaction with a heat source [28,29]. Seddeek studied the thermal radiation and buoyancy effect on MHD free convection heat generation flow over an accelerating permeable surface with the influence temperature dependent viscosity [30], and later the chemical reaction, variable viscosity, radiation, variable suction on hydromagnetic convection flow problems were included [31-33].

Although there are numerous widely practical applications in industrial processes, few previous published papers discussed the combined relation. In the present paper, we make an attempt to investigate the problem of convective heat and mass transfer of incompressible MHD visco-elastic fluid embedded in a porous medium over a stretching sheet under a chemical reaction. The presence of combined buoyancy effects leads to the momentum, heat and mass transfer equations in the coupled form of highly non-linear partial differential equations. To deal with the coupling and non-linearity, a numerical

shooting technique for three unknown initial conditions with Runge-Kutta fourth order integration scheme has been developed. The results are analyzed for various physical parameters such as visco-elasticity, permeability of the porous medium, magnetic field, Grashof number, Schmidt number, Prandtl number, heat source parameter and chemical reaction parameter on the flow, heat and mass transfer characteristics.

2. Mathematical Formulation

We consider a free convective, laminar boundary layer flow and heat and mass transfer of viscous incompressible and electrically conducting visco-elastic liquid due to a stretching sheet. The sheet lies in the plane $y = 0$ with the flow being confined to $y > 0$. The coordinate x is being taken along the stretching sheet and y is normal to the surfaced, two equal and opposite forces are applied along the x-axis, so that the sheet is stretched, keeping the origin fixed. A uniform transverse magnetic field of strength B_0 is applied parallel to the y-axis and the chemical reaction is taking place in the flow. The viscous dissipation effect and Joule heat are neglected on account of the fluid is finitely conducting. It is assumed that the induced magnetic field, the external electric field and the electric field due to the polarization of charges are negligible. The density variation and the effects of the buoyancy are taken into account in the momentum equa- tion (Boussinesq's approximation) and the concentration of species far from the wall is infinitesimally small and the viscous dissipation term in the energy equation is neglected (as the fluid velocity is very low). Under these assumptions, the governing boundary layer equations of momentum, energy and diffusion under Boussinesq approximations could be written as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial \upsilon}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + \upsilon\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} - k_0\left\{u\frac{\partial^3 u}{\partial x \partial y^2} + \upsilon\frac{\partial^3 u}{\partial y^3} + \frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2} - \frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x \partial y}\right\}$$

$$-\frac{\nu}{\varepsilon}u + g\beta(T - T_\infty) + g\beta^*(C - C_\infty) - \frac{\sigma B_0^2}{\rho}u \tag{2}$$

$$u\frac{\partial T}{\partial x} + \upsilon\frac{\partial T}{\partial y} = \frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2} + \frac{Q}{\rho C_p}(T - T_\infty) \tag{3}$$

$$u\frac{\partial C}{\partial x} + \upsilon\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} - K_1(C - C_\infty) \tag{4}$$

where u, υ are velocity components, T and C are, respectively, the temperature and concentration of chemical species in the fluid, ν is the kinematic viscosity, k_0 is the non-Newtonian visco-elastic parameter, ε is

the permeability coefficient of porous medium, g is the acceleration due to gravity, β is the volumetric coefficient of thermal expansion, β^* is the volumetric concentration coefficient, B_0 is the magnetic induction, ρ is the fluid density, σ is the fluid electrical conductivity, k is the thermal conductivity, C_p is the specific heat at constant pressure, Q is the dimensional heat generation/absorption coefficient, D is the mass diffusivity and K_1 is the chemical reaction parameter.

The boundary conditions governing the flow are:

$$y = 0, u = bx, \upsilon = 0, C = C_w + A\left(\frac{x}{l}\right), T = T_w + B\left(\frac{x}{l}\right)$$
$$y \to \infty, u \to \infty, u_y \to \infty, T \to T_\infty, C \to C_\infty$$
$$(5)$$

To take into account the effect of stretching of the boundary sheet, and the effects due to temperature and concentration gradients, we prescribe the wall boundary conditions in the form of (5). In order to study the heat transfer analysis we consider two general cases of non-isothermal temperature boundary conditions, namely boundary with prescribed power law surface temperature. The subscript y denotes the differentiation *w.r.t.* y. Now, we introduce the following dimensionless variables:

$$u = bxf'(\eta), \upsilon = -\sqrt{bv}f(\eta), \eta = \sqrt{\frac{b}{v}}y$$
$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}$$
$$(6)$$

where

$$T_w - T_\infty = B\left(\frac{x}{l}\right), C_w - C_\infty = A\left(\frac{x}{l}\right) \qquad (7)$$

With these changes of variables Equation (1) is identically satisfied and Equations (2)-(4) are transformed to

$$f'^2 - ff'' = f''' - k_1\left\{2f'f''' - ff'''' - f''^2\right\} - k_2 f'$$
$$+ Gr\,\theta + Gc\,\phi - Mf'$$
$$(8)$$

$$\theta'' + Pr\left\{f\theta' - f'\theta - \delta\theta\right\} = 0 \qquad (9)$$

$$\phi'' + Sc\left\{f\phi' - f'\phi + \gamma\phi\right\} = 0 \qquad (10)$$

The corresponding boundary conditions take the form:

$$\eta = 0, f = 0, f' = 1, \theta = 1, \phi = 1$$
$$\eta \to \infty, f' = 0, f' = 0, \theta = 0, \phi = 0$$
$$(11)$$

where subscript ' denotes the differentiation with respect to η. k_1, k_2 are the viscoelastic and porosity parameters, Gr and Gc are the free convection parameters, M magnetic field parameter, δ is the heat generation or absorption coefficient, γ is the Chemical reaction parameter, and Pr, Sc denote Prandtl number and

Schmidt number respectively. These dimensionless physical parameters are defined as:

$$k_1 = \frac{k_0 b}{v}, k_2 = \frac{v}{\varepsilon b}, Gr = \frac{g\beta(T_w - T_\infty)}{b^2 x}, Gc = \frac{g\beta^*(C_w - C_\infty)}{b^2 x},$$
$$Pr = \frac{\mu C_p}{k}, Sc = \frac{v}{D}, \delta = \frac{Q}{b\rho C_p}, \gamma = \frac{K_1}{b} \text{ and } M = \frac{\sigma B_0^2}{\rho b}$$
$$(12)$$

where expressions for $(T_w - T_\infty)$ and $(C_w - C_\infty)$ are given in Equation (6). The important physical quantities of our interest are the local skin friction τ_w, Nusselt number Nu and Sherwood number Sh and they are defined in the sequel:

$$\tau_w = \frac{\tau^*}{\mu bx\sqrt{\frac{b}{v}}} = -f''(0) \text{ where } \tau^* = -\left(\frac{\partial u}{\partial y}\right)_{y=0}$$

$$Nu = -\frac{h}{t_w - T_\infty}T_y = \theta'(0)$$

$$Sh = -\frac{h}{C_w - C_\infty}C_y = \phi'(0)$$

3. Numerical Solution

Equations (8)-(10) constitute a highly non-linear coupled boundary value problem of fourth and second order. So we develop most effective numerical shooting technique with fourth-order Runge-Kutta integration scheme with Newton Raphson method. To select η_∞ we begin with some initial guess value and solve the problem with some particular set of parameters to obtain $f''(0)$, $\theta'(0)$ and $\phi'(0)$. The solution process is repeated with another larger value of η_∞ until two successive values of $f''(0)$, $\theta'(0)$ and $\phi'(0)$ differ only after desired digit signifying the limit of the boundary along η. The last value of η_∞ is chosen as appropriate value for that particular set of parameters.

Equations (8)-(10) of fourth order in f and second order in θ θ and ϕ ϕ has been reduced to a system of eight simultaneous equations of first order for eight unknowns following the method of superposition [34]. To solve this system we require eight initial conditions whilst we have only two initial conditions $f'(0)$ and $f(0)$ on f, two initial conditions on each on θ and ϕ. The third initial condition on $f'''(0)$ has been deduced by applying initial conditions of (11) in Equation (8). Still there are three initial conditions $f''(0)$, $\theta'(0)$ and $\phi'(0)$ which are not prescribed. Now, we employ numerical shooting technique where these two ending boundary conditions are utilized to produce two known initial conditions at $\mu = 0$. In this calculation, the step size $\Delta\eta = 0.001$ is used while obtaining the numerical

solution with $\eta_{\max} = 7$ and five-decimal accuracy as the criterion for convergence.

4. Results and Discussion

The numerical computations have been carried out for various values of visco-elastic parameter k_1, porosity parameter k_2, Grashof number Gr, modified Grashof number Gc, Prandtl number Pr and Schmidt number Sc using numerical scheme discussed in the previous section. In order to illustrate the results graphically, the numerical values are plotted in **Figures 1-18**. These figures depict the horizontal velocity, temperature and concentration profiles for power law surface temperature. Values of local skin friction τ_w, Nusselt number Nu and Sherwood number Sh are recorded in **Table 1-3** for various values of k_1 visco-elastic, k_2 porosity parameter, Gr, Gc free convection parameters, M magnetic field parameter, δ heat generation or absorption coefficient, γ Chemical reaction parameter, Pr Prandtl
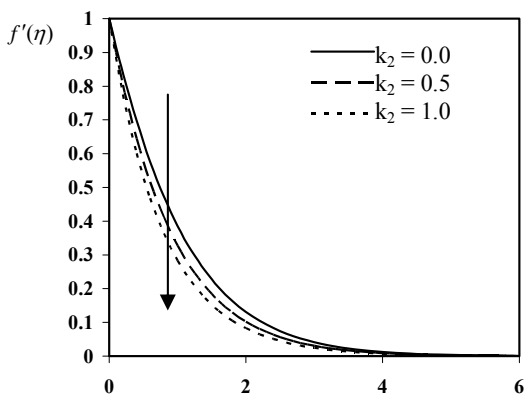
Figure 1. Effect of k_2 on the velocity f'(η) profiles for $k_1 = 0.1$, M = Gr = Gc = 0.5, Pr = 1, Sc = 0.96, $\delta = -0.5$ and $\gamma = 0.5$.
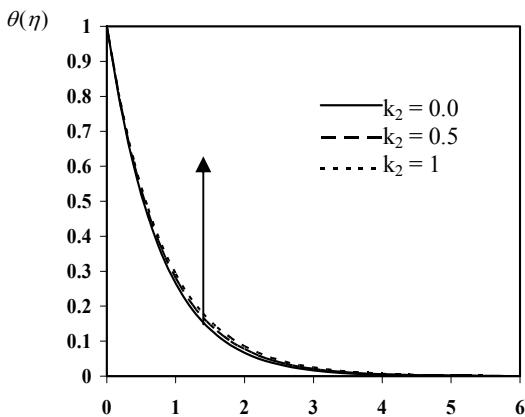
Figure 2. Effect of k_2 on the temperature $\theta(\eta)$ profiles for $k_1 = 0.1$, M = Gr = Gc = 0.5, Pr = 1, Sc = 0.96, $\delta = -0.5$ and $\gamma = 0.5$.
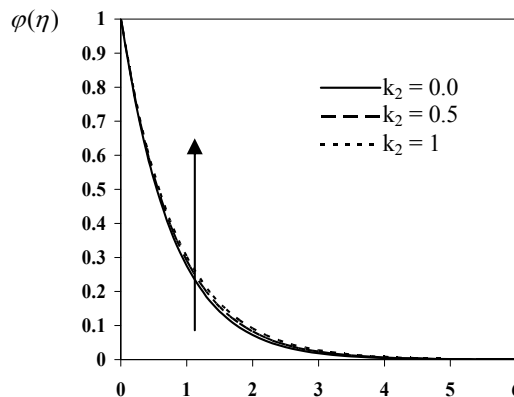

Figure 3. Effect of k_2 on the concentration $\varphi(\eta)$ profiles for $k_1 = 0.1$, M = Gr = Gc = 0.5, Pr = 1, Sc = 0.96 $\delta = -0.5$ and $\gamma = 0.5$.

number and S_c Schmidt number.

Figures 1-3 display results for the velocity, temperature and concentration distributions. As shown, the temperature and concentration are increasing with increasing the dimensionless porous medium parameter k_2 and the velocity decreases as k_2 increases. The effect of the dimensionless porous medium parameter k_2 becomes smaller as k_2 increases.

Figures 4-6 illustrate the influence of the magnetic parameter M on the velocity, temperature and concentration profiles in the boundary layer, respectively. Application of a transverse magnetic field to an electrically conducting fluid gives rise to a resistive-type force called the Lorentz force. This force has the tendency to slow down the motion of the fluid in the boundary layer and to increase its temperature and concentration. Also, the effects on the flow and thermal fields become more so as the strength of the magnetic field increases.

Figures 7-12 show the effects of Grashof number Gr and modified Grashof number Gc on the velocity, temperature and concentration respectively. As shown, the
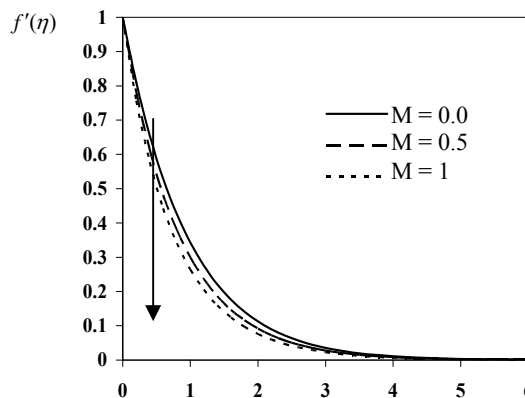
Figure 4. Effect of M on the velocity f'(η) profiles for $k_1 = 0$, $k_2 = 1$, Gr = Gc =0.5, Pr = 1, Sc = 0.96, $\delta = -0.5$ and $\gamma = 0.5$.
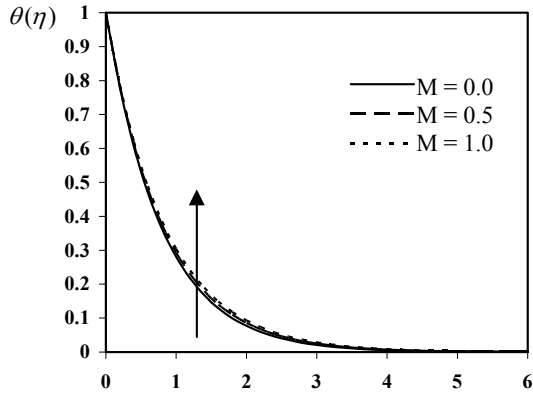
Figure 5. Effect of M on the temperature θ(η) profiles for k₁ = 0.1, k₂ = 1, Gr = Gc = 0.5, Pr = 1, Sc = 0.96, δ = −0.5 and γ = 0.5.

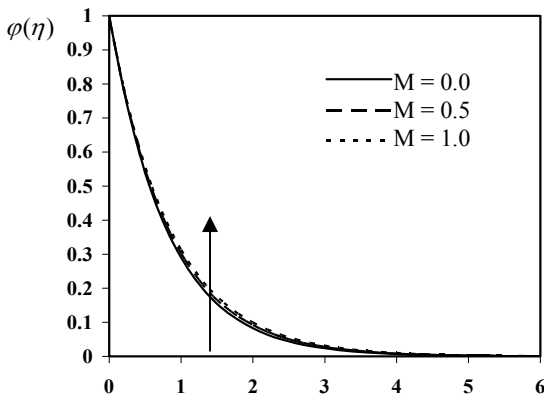**Figure 8. Effect of G_r on the temperature θ(η) profiles for k₁ = 0.1, k₂ = 1, M = Gc = 0.5, P_r= 1, Sc = 0.96, δ = −0.5 and γ = 0.5.**

Figure 6. Effect of Mon the concentration φ(η) profiles for k₁ = 0.1, k₂ = 1, Gr = Gc = 0.5, Pr = 1, Sc = 0.96, δ = −0.5, and γ = 0.5.

Figure 9. Effect of Gr on the concentration φ(η) profiles for k₁ = 0.1, k₂ = 1, M = Gc = 0.5, Pr = 1, Sc = 0.96, δ = −0.5 and γ = 0.5.

Figure 7. effect of Gr on the velocity f'(η) profiles for k₁ = 0.1, k₂ = 1, M = Gc = 0.5, Pr = 1, Sc =0.96, δ = −0.5 and γ = 0.5.

Figure 10. Effect of Gc on the velocity f'(η) profiles for k₁ = 0.1, k₂ = 1, M = Gr = 0.5, Pr = 1, Sc = 0.96, δ = −0.5 and γ = 0.5.

temperature and the concentration are decreasing with increasing *Gr* and *Gc*, but the velocity increases as *Gr*

and *Gc* increases. Physically *Gr* > 0 means heating of the fluid or cooling of the boundary surface, *Gr* > 0

Figure 11. effect of Gc on the temperature θ(η) profiles fvor k₁ = 0.1, k₂ = 1, M = Gr = 0.5, Pr = 1, Sc = 0.96, δ = −0.5 and γ = 0.5.
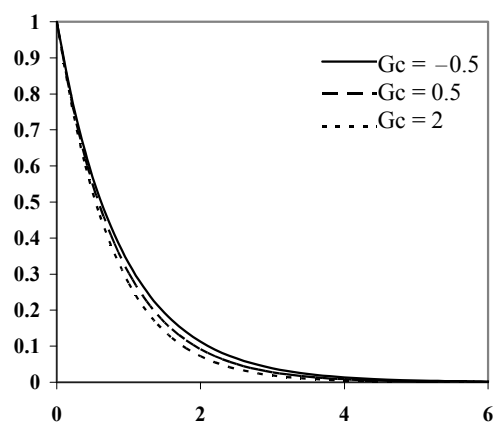
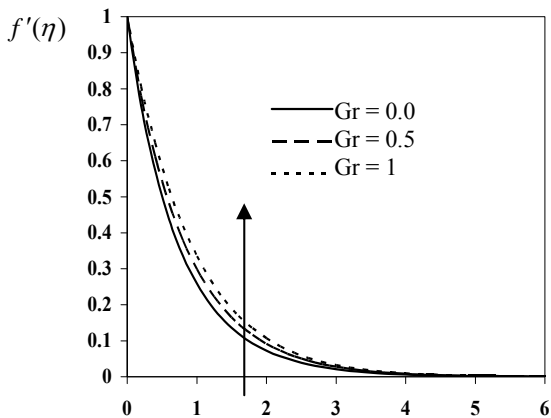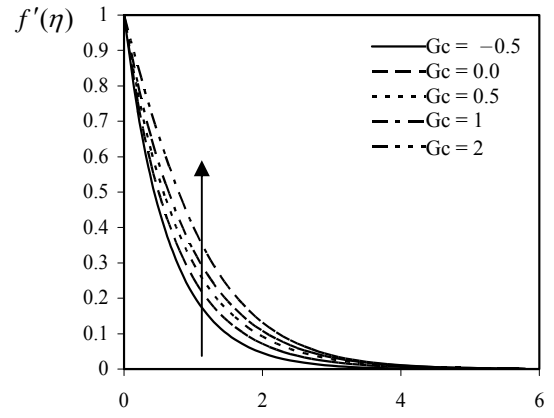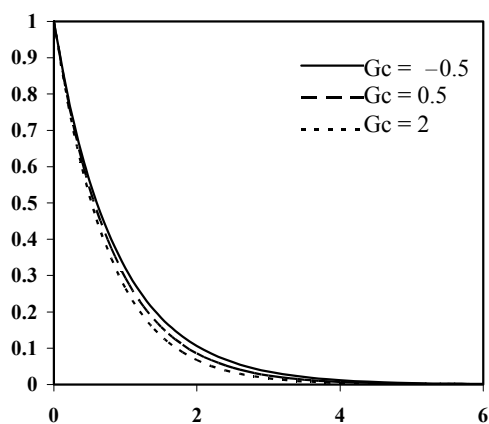**Figure 12. Effect of Gc on the concentration φ(η) profiles for k₁ = 0.1, k₂ =1, M = Gr = 0.5, Pr = 1, S_c = 0.96, δ = −0.5 and γ = 0.5.**
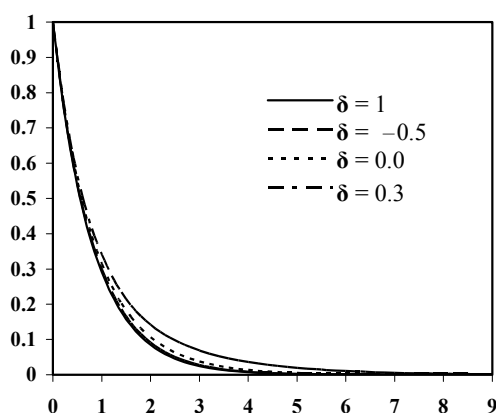
Figure 13. Effect of δ on the velocity f′(η) profiles for k₁ = 0.1, k₂ = 1, M = Gr = Gc = 0.5, Pr = 1, Sc = 0.96 and γ = 0.5.

means cooling of the fluid or heating of the boundary surface and $Gr = 0$ corresponds to the absence of free con-

Figure 14. Effect of δ on the temperature θ(η) profiles for k₁ = 0.1, k₂ = 1, M = Gr = Gc = 0.5, Pr = 1, Sc = 0.96 and γ = 0.5.

Figure 15. Effect of δ on the concentration φ(η) profiles for k₁ = 0.1, k₂ = 1, M = Gr = Gc = 0.5, Pr = 1, Sc = 0.96 and γ = 0.5.

vection current.

Figures 13-15 present typical profiles for the velocity, temperature and concentration for various values of a heat source ($\delta > 0$) or a heat sink ($\delta < 0$), respectively. As shown, the velocity and the temperature are increasing with increasing δ, but the concentration decreases as δ increases. In the event that the strength of the heat sink ($\delta < 0$) is relatively large, the maximum fluid temperature does not occur at the wall but rather in the fluid region close to it. Conversely, the presence of a heat source ($\delta > 0$) effect causes a reduction in the thermal state of the fluid, thus producing lower thermal boundary layers.

Figures 16-18 illustrate the influence of the Chemical reaction parameter γ on the velocity, temperature and concentration profiles in the boundary layer, respectively. As shown, the velocity and the concentration are decreasing with increasing δ, but the temperature increases

Figure 16. Effect of γ on the velocity f'(η) profiles for k_1 = 0.1, k_2 = 1, M = Gr = Gc = 0.5, Pr = 1, Sc = 0.96 and δ = –0.5.

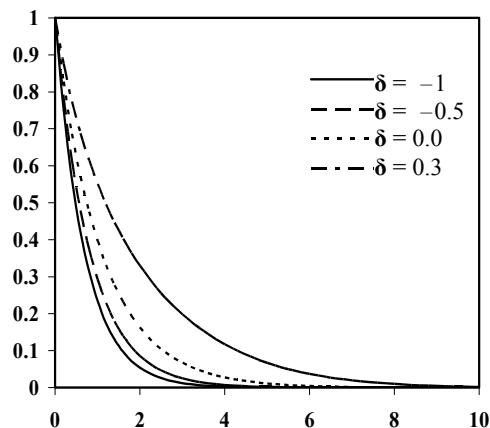**Figure 18. Effect of γ on the concentration φ(η) profiles for k_1 = 0.1, k_2 = 1, M = Gr = Gc = 0.5, Pr = 1, Sc = 0.96 and δ = –0.5.**

Figure 17. Effect of γ on the temperature θ(η) profiles for k_1 = 0.1, k_2 = 1, M = Gr = Gc = 0.5, Pr = 1, Sc = 0.96 and δ = –0.5.

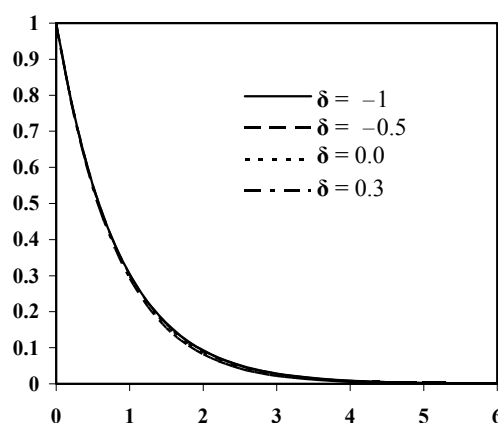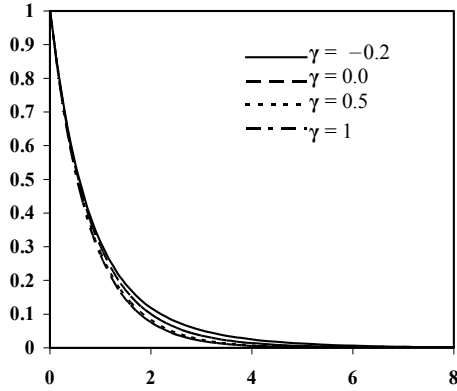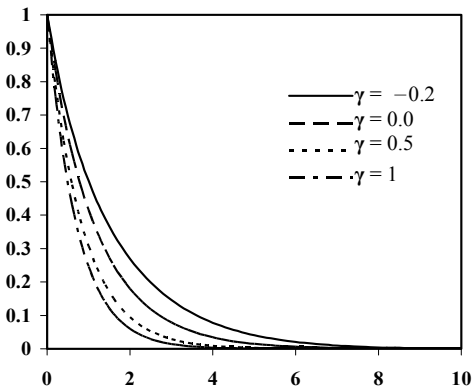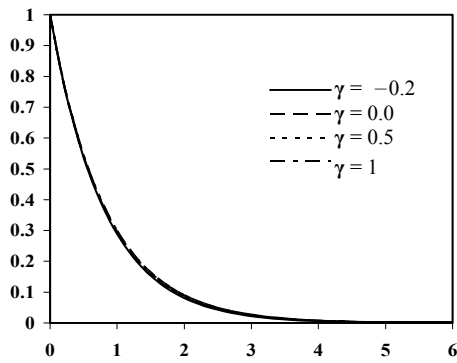as δ increases, this is due to the fact that destructive chemical reduces the solutal boundary layer thickness and increases the mass transfer.

Tables 1-3 represents values of τ_w, Nu and Sh for various values of δ, γ, M, Gr, Gc, Pr, Sc, k_1 and k_2. It is clear that, with increasing Gr and Gc, Nu, Sh and τ_w increases, whereas with increasing k_1, k_2 and M, Nu, Sh and τ_w decreases. Also, τ_w and Sh increase as Sc and γ increases and Nu decrease, whereas, Nu and τ_w decreases and Sh increase as δ increases, Also, Sh and τ_w increases and Nu decrease as Pr increase.

Table 1. The effect of parameter on $f''(0)$.

					$f''(0)$						
$k_1 = 0.3$	$k_1 = 0.2$	$k_1 = 0.1$	k_2	M	Gr	Gc	Pr	δ	Sc	γ
–0.9233	–0.8908	–0.8626	0.0	0.5	0.5	0.5	1.0	–0.5	0.96	0.5
–1.1978	–1.1344	–1.0838	0.5	0.5	0.5	0.5	1.0	–0.5	0.96	0.5
–1.4345	–1.3476	–1.2791	1.0	0.5	0.5	0.5	1.0	–0.5	0.96	0.5
–1.1978	–1.1344	–1.0838	1.0	0.0	0.5	0.5	1.0	–0.5	0.96	0.5
–1.4345	–1.3476	–1.2791	1.0	0.5	0.5	0.5	1.0	–0.5	0.96	0.5
–1.6446	–1.5387	–1.4552	1.0	1.0	0.5	0.5	1.0	–0.5	0.96	0.5
–1.6639	–1.556	–1.4692	1.0	0.5	0.0	0.5	1.0	–0.5	0.96	0.5
–1.4345	–1.3476	–1.2791	1.0	0.5	0.5	0.5	1.0	–0.5	0.96	0.5
–1.1994	–1.1400	–1.0929	1.0	0.5	1.0	0.5	1.0	–0.5	0.96	0.5
–1.8913	–1.7696	–1.6688	1.0	0.5	0.5	–0.5	1.0	–0.5	0.96	0.5
–1.6654	–1.5578	–1.4712	1.0	0.5	0.5	0.0	1.0	–0.5	0.96	0.5
–1.4345	–1.3476	–1.2791	1.0	0.5	0.5	0.5	1.0	–0.5	0.96	0.5
–1.4345	–1.3476	–1.2791	1.0	0.5	0.5	0.5	1.0	–0.5	0.96	0.5
–1.4622	–1.379	–1.3120	1.0	0.5	0.5	0.5	2.0	–0.5	0.96	0.5
–1.5010	–1.4201	–1.3531	1.0	0.5	0.5	0.5	5.5	–0.5	0.96	0.5
–1.4457	–1.3602	–1.2922	1.0	0.5	0.5	0.5	1.0	–1.0	0.96	0.5
–1.4345	–1.3476	–1.2791	1.0	0.5	0.5	0.5	1.0	–0.5	0.96	0.5
–1.4165	–1.3268	–1.2571	1.0	0.5	0.5	0.5	1.0	0.0	0.96	0.5
–1.4034	–1.3110	–1.2402	1.0	0.5	0.5	0.5	1.0	0.2	0.96	0.5
–1.4345	–1.3476	–1.2791	1.0	0.5	0.5	0.5	1.0	–0.5	0.96	0.5
–1.4637	–1.3808	–1.3140	1.0	0.5	0.5	0.5	1.0	–0.5	2.0	0.5
–1.5025	–1.4219	–1.3551	1.0	0.5	0.5	0.5	1.0	–0.5	5.0	0.5
–1.4032	–1.3106	–1.2397	1.0	0.5	0.5	0.5	1.0	–0.5	0.96	–0.2
–1.4164	–1.3267	–1.2569	1.0	0.5	0.5	0.5	1.0	–0.5	0.96	0.0
–1.4345	–1.3476	–1.2791	1.0	0.5	0.5	0.5	1.0	–0.5	0.96	0.5
–1.4457	–1.3603	–1.2923	1.0	0.5	0.5	0.5	1.0	–0.5	0.96	1.0

Table 2. The effect of parameter on $-\theta'(0)$.

$-\theta'(0)$										
$k_1 = 0.3$	$k_1 = 0.2$	$k_1 = 0.1$	k_2	M	Gr	Gc	Pr	δ	Sc	γ
1.2609	1.2663	1.2711	0.0	0.5	0.5	0.5	1.0	−0.5	0.96	0.5
1.2282	1.2360	1.2425	0.5	0.5	0.5	0.5	1.0	−0.5	0.96	0.5
1.2013	1.2106	1.2184	1.0	0.5	0.5	0.5	1.0	−0.5	0.96	0.5
1.2282	1.2360	1.2425	1.0	0.0	0.5	0.5	1.0	−0.5	0.96	0.5
1.2013	1.2106	1.2184	1.0	0.5	0.5	0.5	1.0	−0.5	0.96	0.5
1.1785	1.1889	1.1976	1.0	1.0	0.5	0.5	1.0	−0.5	0.96	0.5
1.1724	1.1841	1.1941	1.0	0.5	0.0	0.5	1.0	−0.5	0.96	0.5
1.2013	1.2106	1.2184	1.0	0.5	0.5	0.5	1.0	−0.5	0.96	0.5
1.2278	1.2346	1.2404	1.0	0.5	1.0	0.5	1.0	−0.5	0.96	0.5
1.1390	1.1529	1.1650	1.0	0.5	0.5	−0.5	1.0	−0.5	0.96	0.5
1.1719	1.1835	1.1935	1.0	0.5	0.5	0.0	1.0	−0.5	0.96	0.5
1.2013	1.2106	1.2184	1.0	0.5	0.5	0.5	1.0	−0.5	0.96	0.5
1.2013	1.2106	1.2184	1.0	0.5	0.5	0.5	1.0	−0.5	0.96	0.5
1.7734	1.7849	1.7946	1.0	0.5	0.5	0.5	2.0	−0.5	0.96	0.5
2.9327	2.9463	2.9578	1.0	0.5	0.5	0.5	5.5	−0.5	0.96	0.5
1.4134	1.4207	1.4268	1.0	0.5	0.5	0.5	1.0	−1.0	0.96	0.5
1.2013	1.2106	1.2184	1.0	0.5	0.5	0.5	1.0	−0.5	0.96	0.5
0.9181	0.9321	0.9442	1.0	0.5	0.5	0.5	1.0	0.0	0.96	0.5
0.7494	0.7656	0.7827	1.0	0.5	0.5	0.5	1.0	0.2	0.96	0.5
1.2013	1.2106	1.2184	1.0	0.5	0.5	0.5	1.0	−0.5	0.96	0.5
1.1922	1.2016	1.2097	1.0	0.5	0.5	0.5	1.0	−0.5	2.0	0.5
1.1836	1.1934	1.2020	1.0	0.5	0.5	0.5	1.0	−0.5	5.0	0.5
1.2141	1.2232	1.2305	1.0	0.5	0.5	0.5	1.0	−0.5	0.96	−0.2
1.2083	1.2174	1.2249	1.0	0.5	0.5	0.5	1.0	−0.5	0.96	0.0
1.2013	1.2106	1.2184	1.0	0.5	0.5	0.5	1.0	−0.5	0.96	0.5
1.1975	1.2069	1.2149	1.0	0.5	0.5	0.5	1.0	−0.5	0.96	1.0

Table 3. The effect of parameter on $-\phi'(0)$.

$-\phi'(0)$										
$k_1 = 0.3$	$k_1 = 0.2$	$k_1 = 0.1$	k_2	M	Gr	Gc	Pr	δ	Sc	γ
1.2322	1.2375	1.2423	0.0	0.5	0.5	0.5	1.0	−0.5	0.96	0.5
1.1999	1.2075	1.2140	0.5	0.5	0.5	0.5	1.0	−0.5	0.96	0.5
1.1733	1.1824	1.1902	1.0	0.5	0.5	0.5	1.0	−0.5	0.96	0.5
1.1999	1.2075	1.2140	1.0	0.0	0.5	0.5	1.0	−0.5	0.96	0.5
1.1733	1.1824	1.1902	1.0	0.5	0.5	0.5	1.0	−0.5	0.96	0.5
1.1508	1.1610	1.1697	1.0	1.0	0.5	0.5	1.0	−0.5	0.96	0.5
1.1448	1.1563	1.1661	1.0	0.5	0.0	0.5	1.0	−0.5	0.96	0.5
1.1733	1.1824	1.1902	1.0	0.5	0.5	0.5	1.0	−0.5	0.96	0.5
1.1994	1.2062	1.2119	1.0	0.5	1.0	0.5	1.0	−0.5	0.96	0.5
1.1118	1.1255	1.1374	1.0	0.5	0.5	-0.5	1.0	−0.5	0.96	0.5
1.1442	1.1557	1.1655	1.0	0.5	0.5	0.0	1.0	−0.5	0.96	0.5
1.1733	1.1824	1.1902	1.0	0.5	0.5	0.5	1.0	−0.5	0.96	0.5
1.1733	1.1824	1.1902	1.0	0.5	0.5	0.5	1.0	−0.5	0.96	0.5
1.1648	1.1741	1.1821	1.0	0.5	0.5	0.5	2.0	−0.5	0.96	0.5
1.1563	1.1660	1.1744	1.0	0.5	0.5	0.5	5.5	−0.5	0.96	0.5
1.1696	1.1789	1.1868	1.0	0.5	0.5	0.5	1.0	−1.0	0.96	0.5
1.1733	1.1824	1.1902	1.0	0.5	0.5	0.5	1.0	−0.5	0.96	0.5
1.1801	1.1891	1.1965	1.0	0.5	0.5	0.5	1.0	0.0	0.96	0.5
1.1858	1.1948	1.2020	1.0	0.5	0.5	0.5	1.0	0.2	0.96	0.5
1.1733	1.1824	1.1902	1.0	0.5	0.5	0.5	1.0	−0.5	0.96	0.5
1.7729	1.7844	1.7940	1.0	0.5	0.5	0.5	1.0	−0.5	2.0	0.5
2.9322	2.9458	2.9573	1.0	0.5	0.5	0.5	1.0	−0.5	5.0	0.5
0.7266	0.7420	0.7589	1.0	0.5	0.5	0.5	1.0	−0.5	0.96	−0.2
0.8935	0.9194	0.9194	1.0	0.5	0.5	0.5	1.0	−0.5	0.96	0.0
1.1733	1.1824	1.1902	1.0	0.5	0.5	0.5	1.0	−0.5	0.96	0.5
1.3820	1.3891	1.3952	1.0	0.5	0.5	0.5	1.0	−0.5	0.96	1.0

5. Conclusions

In this study, a numerical analysis is presented to investigate the influence of chemical reaction of first-order and magnetic field on the heat and mass transfer of an electrically conducting viscoelastic fluid flow through a porous medium over a stretching sheet. The non-linear and coupled governing equations are solved numerical by using fourth order Runge-Kutta integration scheme-with Newton Raphson shooting method. Velocity, tem-

perature and concentration profiles are presented graphically and analyzed. The fundamental parameters found to effect the problem under consideration are the chemical reaction parameter, magnetic field parameter, viscoelastic parameter, porosity parameter, Grashof number, modified Grashof number, Prandtl number, Schmidt number and heat absorption parameter. It is found that, the temperature as well as concentration increases with increasing the visco-elastic parameter, porosity parameter and magnetic parameter whereas reverse trend is seen with Grashof number and modified Grashof number increasing. Additionally, the velocity temperature is increased in the presence of heat absorption parameters and decreased with chemical reaction.

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Nomenclature

A, B	constants	T_∞	temperature of the ambient fluid
b	stretching rate, positive constant	T_w	surface temperature
B_0	magnetic induction	u, υ	velocity components along x and y direction
C_p	specific heat at constant pressure	x	coordinate along the stretching sheet
D	mass diffusivity	y	distance normal to the stretching sheet
f	dimensionless stream function	*Greek symbols*	
Gr	temperature Grashof of number	θ	dimensionless temperature function
Gc	mass Grashof of number	ϕ	dimensionless concentration function
g	acceleration due to gravity	η	dimensionless space variable
K_1	first order chemical reaction rate	ν	kinematic viscosity
k	thermal conductivity	ρ	fluid density
k_0	non-Newtonian visco-elastic parameter	σ	electrical conductivity
k_1	visco-elastic parameter	μ	coefficient of viscosity
k_2	porosity parameter	β	coefficient of thermal expansion
l	characteristic length	β^*	volumetric concentration coefficient
M	magnetic field parameter	δ	heat generation or absorption coefficient
Nu	Nusselt number	τ_w	skin friction
Pr	Prandtl number	ε	permeability coefficient of porous medium
Q	dimensional heat generation/absorption coefficient	γ	Chemical reaction parameter
Sc	Schmidt number	*Subscripts*	
Sh	Sherwood number	w	properties at the plate
T	temperature of the fluid	∞	free stream condition