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Controllability of Complex Power Networks

Guohua Zhang[1], Zhen Li[1] & Qiaoli Zhang[1]

[1] Hunan University of Technology, China

Correspondence: Guohua Zhang, Hunan University of Technology, Zhuzhou, Hunan, 412000, China. E-mail: 9884188@qq.com

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Abstract

With the progress of time, the power network has been the basis of economic development. However, people have little knowledge of the controllability of the power network. This article will study eight power networks and compare the controllability of the power network in many aspects.

Keywords: power network, controllable, command

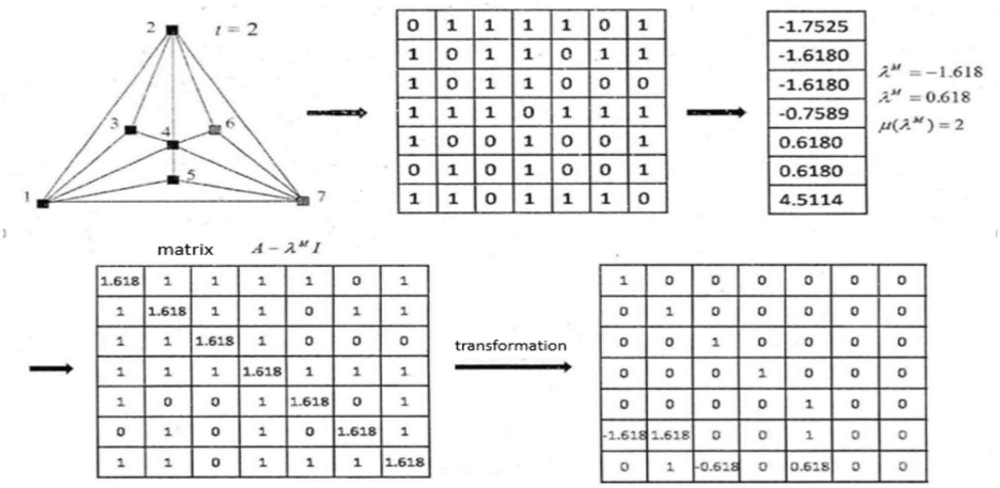
1. Introduction

Power network supports the development of modern society, almost all industrial and life are highly dependent on electricity. China's power network covers an area of about 88% of the land area. Power transmission over an extremely long distance has caused the analysis of the dynamic behavior of power transmission in the network is rather complicated. The traditional method has shown its limitations.

Last few years, professional researchers began to study from the complex power network, the main study of the topology analysis, the robustness of the power network and attack vulnerabilities, power cascades and other cascading failure mechanism. However, few studies have connected the controllability of the network. In this paper, we use the controllability theory of the network to analyze the controllability of the network, the research objects include four node systems of IEEE standard test system, node system of North-East China (NEC) grid and node system of Middle China (MC), NEC and MC will adopt the connections and nodes above 220kv for modeling. Firstly, the controllability of the eight-power network is studied and compared with the controllability of ER stochastic networks, WS small world networks, NW small world networks, BA scale-free networks, configuration model (CM) After comparing, the influence of the main structural characteristics of power network on the controllability is also studied.

2. Methods and models

Use controllability theory to research the grid, how to solve the controllability theory (Figure 1).



Coupling matrix A

0	1	1	1	1	0	1
1	0	1	1	0	1	1
1	0	1	1	0	0	0
1	1	1	0	1	1	1
1	0	0	1	0	0	1
0	1	0	1	0	0	1
1	1	0	1	1	1	0

Eigenvalues λ

-1.7525
-1.6180
-1.6180
-0.7589
0.6180
0.6180
4.5114

matrix $A - \lambda^M I$

1.618	1	1	1	1	0	1
1	1.618	1	1	0	1	1
1	1	1.618	1	0	0	0
1	1	1	1.618	1	1	1
1	0	0	1	1.618	0	1
0	1	0	1	0	1.618	1
1	1	0	1	1	1	1.618

1	0	0	0	0	0	0
0	1	0	0	0	0	0
0	0	1	0	0	0	0
0	0	0	1	0	0	0
0	0	0	0	1	0	0
-1.618	1.618	0	0	1	0	0
0	1	-0.618	0	0.618	0	0

Figure 1. Network controllable node calculation

Network configuration model (CM) [1]is an extension of the random network model ER, and ER model except that the model may be distributed P (k) according to the degree of power network generates a random, construct algorithm 2.1.

Algorithm 2.1:

```
Input: Degree distribution P (k)
Output: A network having a random distribution of P (k) of
1). Generate a degree sequence of length N {k1, k2, k3 ..., $k_N$} according to the degree distribution P (k), representing degrees of N nodes of the network;
2). Select two nodes 1 and j ($k_i$, $k_j$> 0) randomly from the network, and subtract 1 from the corresponding degree values $k_i$ and $k_j$, respectively;
3). Repeat step 2 until all nodes in the network have a value of 0; return to the resulting network.
```

Network configuration model has the following mathematical properties:

(1). The probability P_{ij} of edge connection exists between any two nodes i, j in the network. If two nodes are connected according to the random matching algorithm (Algorithm 2.1), one branch on the "Stump" corresponding to node i is connected to node j with a probability of k_j / (2L-1),while the node i itself has k_i branches, so the probability that the node i, j has a connection P_{ij} is

$$p_{ij} = \frac{k_i k_j}{2L-1} \approx \frac{k_i k_j}{2L} \tag{2.1}$$

(2).The above formula when the number of network edge L is sufficiently large, the second equality holds. Equation (2.1) clearly shows that the greater the degree of nodes i, j, the greater the probability of their connection in the configuration network model. In addition, other important indicators of network configuration model also includes the number of edges parallel to the desired network

$$E(pallel-edges) = \frac{1}{2}\left[\frac{<k^2>-<k>}{<k>}\right]^2 \tag{2.2}$$

(3).The number of self-loop network expectations

$$E(self-loops) = \frac{<k^2>-<k>}{2<k>} \tag{2.3}$$

(4). Network aggregation coefficient

$$C = \frac{1}{N}\frac{(<k^2>-<k>)^2}{<k>^3} \tag{2.4}$$

3. Numerical simulation results

Six power network IEEE paper: 30,57, 118, 145, 162, 300, NEC 127 and MC 302 node grid. For comparison, the same scale WS small-world network, ER stochastic network, CM configuration model network and BA scale-free network are set up. The CM configuration model network has the same degree distribution as the power network.

The main characteristics of the electricity network topology including nodes N, the number of edges L, the average degree of <k>, clustering coefficient C, the diameter D and the average path length APL. As shown in Table 1:

Table 1.

Network	N	L	<k>	C	D	APL
IEEE 30	30	41	2.733	0.261	6	3.306
IEEE 57	57	78	2.737	0.124	12	4.954
IEEE 118	118	179	3.034	0.175	14	6.309
IEEE 145	145	422	5.821	0.581	11	4.391
IEEE 162	162	280	3.457	0.106	12	5.657
IEEE 300	300	409	2.727	0.111	24	9.935
NEC 127	127	163	2.567	0.071	16	7.112
MC 302	302	396	2.623	0.167	33	12.908
WS 30	30	60	4.000	0.363	5	2.871
WS 57	57	114	4.000	0.253	7	3.365
WS 118	118	236	4.000	0.262	9	4.441
WS 145	145	290	4.000	0.387	11	5.562
WS 162	162	324	4.000	0.341	10	5.287
WS 300	300	600	4.000	0.251	10	5.306
WS 127	127	254	4.000	0.292	9	4.664
WS 302	302	604	4.000	0.286	11	5.573
ER 30	30	41	2.733	0.133	9	3.175
ER 57	57	78	2.737	0.028	12	4.479
ER 118	118	181	3.068	0.008	9	4.323
ER 145	145	412	5.683	0.0345	6	3.026
ER 162	162	283	3.494	0.0108	INF	INF
ER 300	300	406	2.707	0.003	INF	INF
ER 127	127	178	2.803	0.009	10	4.785
ER 302	302	393	2.603	0.008	INF	INF
CM 30	30	41	2.733	0.0222	9	3.526
CM 57	57	78	2.737	0.045	10	4.135
CM 118	118	179	3.034	0.209	10	4.556
CM 145	145	422	5.821	0.0956	7	3.081
CM 162	162	280	3.457	0.0155	9	4.312
CM 300	300	409	2.727	0.0095	13	5.969
CM 127	127	163	2.567	0.0037	13	5.574
CM 302	302	396	2.623	0.0169	15	6.004
BA 30	30	56	3.733	0.172	5	2.506
BA 57	57	110	3.860	0.182	5	2.718
BA 118	118	232	3.932	0.152	5	2.929
BA 145	145	286	3.945	0.0966	5	3.1156
BA 162	162	320	3.951	0.0843	6	3.284
BA 300	300	596	3.973	0.076	7	3.602
BA 127	127	250	3.937	0.127	6	3.110
BA 302	302	600	3.974	0.056	7	3.507

It can be seen from Table 1 that the small-world network contains most of the power grids. (1) WS small- world network path length and the average path length of the gap is small, with the characteristics of a small world; (2) their clustering coefficient is much higher than the corresponding ER random network, with high concentration characteristics.

Calculate the controllability of the power network N_D, Figure 2:

	IEEE30	IEEE57	IEEE145	IEEE162	IEEE118	IEEE300	NEC127	MC302
🟦 N_D	0.0667	0.0175	0.1103	0.0123	0.2119	0.1000	0.1024	0.1060
🟥 ER'sN_D	0.0667	0.0877	0.0069	0.0432	0.0593	0.1100	0.0157	0.1258
🟩 WS'sN_D	0.0333	0.0175	0.0069	0.0062	0.0085	0.0033	0.0079	0.0033
🟪 BA'sN_D	0.0667	0.1052	0.1724	0.1234	0.1525	0.1333	0.1339	0.1060
🟦 CM'sN_D	0.0667	0.0175	0.0138	0.0123	0.0169	0.0533	0.0787	0.0596

Figure 2. Controllability of Power Network N_D Compared with the controllability of the same network

It can be seen from Figure 2 that the controllability N_D of IEEE118, IEEE145, and NEC127 networks is much larger than that of the corresponding ER networks, indicating that ER networks are easier to control than them. The controllability of the power network N_D is greater than or equal to the corresponding CM network, indicating that there are other factors that determine the network controllability in addition to the degree distribution,because the power network has the same degree distribution as the CM network,if the degree distribution is the only factor that determines the controllability of the network, the controllability N_D of the power network and the controllability N_D of the CM network should be exactly the same.There are practical difference between the two, the reason may be attributed to a large APL power network (Table 1),because the actual power network in the design needs to consider the geographical location of the power station and high-voltage line length and other physical constraints.

4. Conclusion

In this paper, the controllability of six kinds of IEEE power networks, Middle China power network and North-East China power network is studied by using controllability theory. It is found that the degree distribution of power network obeys power law distribution and exhibits scale-free characteristic. The controllable value N_D of electric network IEEE 118, IEEE 145 and Northeast China power network is much higher than that of corresponding ER random network, so it is more difficult to control; while other IEEE networks and China Central China power network are easier to control.

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