British Journal of Mathematics & Computer Science
11(1): 1-8, 2015, Article no.BJMCS.19525
ISSN: 2231-0851

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MOND Kinematics of n-Pendulums

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Article Information

DOI: 10.9734/BJMCS/2015/19525
Editor(s):
(1) Dijana Mosi´c, Department of Mathematics, University of Niš, Serbia.
Reviewers:
(1) Ayhan Goktepe, Selcuk University, Konya, Turkey.
(2) Anonymous, Mexico.
Complete Peer review History: http://sciencedomain.org/review-history/10604

Original Research Article

Received: 14 June 2015
Accepted: 22 July 2015
Published: 19 August 2015

Abstract

We generalized our earlier result (Nahatkar and Pund [6]) with regard to the motion of a simple pendulum in a plane.

Keywords: MOND; Kinematics; n-Pendulums; Kinetic energy; Euler-Lagrange equation.

1 Introduction

In an attempt to explain the observed uniform velocities of galaxies, Professor Milgrom in 1983 [3-5] propounded an equation of motion which resulted in the postulation of a theory known as MOND (Modified Newtonian Dynamics). In the proposed theory Newton's second law of motion,

$$\boldsymbol{F} = m\,\boldsymbol{a} \tag{1}$$

is generalized as

$$\boldsymbol{F} = m\,\mu\left(\frac{a}{a_0}\right)\boldsymbol{a}, \tag{2}$$

where μ is an interpolation function given by

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$$\mu\left(\frac{a}{a_0}\right)=\begin{cases}\frac{a}{a_0} & a \ll a_0 \\ 1 & a \gg a_0\end{cases}.$$

For $a \gg a_0$, above equation (2) reduces to (1). The quantity a_0 is a constant having the dimensions of an acceleration a and is evaluated as $a_0 = 1\cdot 2\times 10^{-8}\, cm\,\sec^{-2}$ (Milgrom and Bekenstein) [1-5].

Very recently adopting the form of kinetic energy proposed by Pankovic and Kapor [7]:

$$T(v,a)=\frac{mv^2}{2}\frac{a}{a+a_0}=\frac{mv^2}{2}\frac{\dot{v}}{\dot{v}+a_0}, \tag{3}$$

we (Nahatkar and Pund [6]), have claimed that the Newtonian analogue can be restored in the study of a simple pendulum.

In view of this finding we intend to generalize the result to a system consisting of multiple pendulums i.e. a system of more than a single pendulum.

Sections-2 and 3 are devoted to the study of two-pendulums and three-pendulums respectively. In Section-4 this work is generalized to a system of n-pendulums.

2 System of Two-Pendulums in MOND

Consider a system of two-pendulums of masses $m_1(x_1, y_1)$ and $m_2(x_2, y_2)$ with the effective lengths l_1 and l_2. For simplicity let, $x_2 = k_1 x_1$ and $y_2 = k_1 y_1$, where k_1 is positive constant. Then the kinetic energy of the system in Newtonian dynamics can be expressed as

$$T=\frac{1}{2}l_1^{\,2}\dot{\theta}_1^{\,2}[m_1+m_2k_1^{\,2}] \tag{4}$$

where θ_1 be the angular displacement of the system of two-pendulums from the equilibrium position.Using equation (3), the MOND analogue of the above assumes the form

$$T=\frac{l_1^{\,3}\ddot{\theta}_1\dot{\theta}_1^{\,2}}{2}\left[\frac{m_1}{l_1\ddot{\theta}_1+a_0}+\frac{k_1^{\,3}m_2}{k_1l_1\ddot{\theta}_1+a_0}\right]. \tag{5}$$

Then the Lagrangian L of the system is

$$L=\frac{l_1^{\,3}\ddot{\theta}_1\dot{\theta}_1^{\,2}}{2}\left[\frac{m_1}{l_1\ddot{\theta}_1+a_0}+\frac{k_1^{\,3}m_2}{k_1l_1\ddot{\theta}_1+a_0}\right]-V(\theta_1) \tag{6}$$

Since $L = L(\theta_1, \dot{\theta}_1, \ddot{\theta}_1)$, it satisfies the generalized Euler-Lagrange equation

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_1}\right) - \frac{d^2}{dt^2}\left(\frac{\partial L}{\partial \ddot{\theta}_1}\right) = -\frac{\partial V}{\partial \theta_1} \tag{7}$$

Now

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_1}\right) = \frac{m_1 l_1^3 \ddot{\theta}_1^{\,2}}{l_1\ddot{\theta}_1 + a_0} + \frac{m_2 k_1^{\,3} l_1^3 \ddot{\theta}_1^{\,2}}{k_1 l_1 \ddot{\theta}_1 + a_0} + O_1 \tag{8}$$

and

$$\frac{d^2}{dt^2}\left(\frac{\partial L}{\partial \ddot{\theta}_1}\right) = \frac{m_1 l_1^{\,3} a_0 \ddot{\theta}_1^{\,2}}{(l_1\ddot{\theta}_1 + a_0)^2} + \frac{m_2 k_1^{\,3} l_1^{\,3} a_0 \ddot{\theta}_1^{\,2}}{(k_1 l_1 \ddot{\theta}_1 + a_0)^2} + O_2\,. \tag{9}$$

The first two terms on the right sides of (8) and (9) constitute the leading terms, while O_1 and O_2 represent non-leading terms which are not significant to our result. With these approximations, the Euler-Lagrange equation (7) yields,

$$\frac{m_1 l_1^{\,3} \ddot{\theta}_1^{\,2}}{(l_1\ddot{\theta}_1 + a_0)}\left[1 - \frac{a_0}{l_1\ddot{\theta}_1 + a_0}\right] + \frac{m_2 k_1^{\,3} l_1^{\,3} \ddot{\theta}_1^{\,2}}{(k_1 l_1 \ddot{\theta}_1 + a_0)}\left[1 - \frac{a_0}{k_1 l_1 \ddot{\theta}_1 + a_0}\right] = -\frac{\partial V}{\partial \theta_1} \tag{10}$$

Especially for $l_1\ddot{\theta}_1 >> a_0$ i.e. for Newtonian regime (10) turns out approximately to

$$(m_1 + k_1^{\,2} m_2)\, l_1^2\, \ddot{\theta}_1 \approx -\frac{\partial V}{\partial \theta_1} \tag{11}$$

and for $l_1\ddot{\theta}_1 << a_0$ i.e. for MOND, we have

$$(m_1 + k_1^{\,3} m_2)\, l_1^3\, \ddot{\theta}_1^{\,2} / a_0 \approx -\frac{\partial V}{\partial \theta_1}\,. \tag{12}$$

Since the potential energy V is arbitrary, scaling it to $\frac{V}{l}$ and restoring the symbol V for the new scaled quantity $\frac{V}{l}$, equations (11) and (12) respectively become

$$(m_1 + k_1^{\,2} m_2)\, l_1\, \ddot{\theta}_1 \approx -\frac{\partial V}{\partial \theta_1}\,, \tag{13}$$

and

$$(m_1 + k_1^{\,3} m_2)\, l_1^2\, \ddot{\theta}_1^{\,2} / a_0 \approx -\frac{\partial V}{\partial \theta_1}\,. \tag{14}$$

The MOND inference (14) is obviously different from its Newtonian counterpart (13).

3 System of Three-Pendulums in MOND

Now consider pendulums of masses $m_1(x_1, y_1)$, $m_2(x_2, y_2)$, $m_3(x_3, y_3)$ with the effective lengths l_1, l_2 and l_3. Assuming $x_2 = k_1 x_1$ and $y_2 = k_1 y_1$, $x_3 = k_2 x_1$ and $y_3 = k_2 y_1$, where k_1, k_2 are positive constants, the kinetic energy of the system in Newtonian dynamics is given by

$$T = \frac{1}{2} l_1^2 \dot{\theta}_1^2 \left[m_1 + m_2 k_1^2 + m_3 k_2^2 \right], \tag{15}$$

where θ_1 be the angular displacement of the total system of three-pendulums from the equilibrium position. Using equation (3), we obtain

$$T = \frac{l_1^3 \ddot{\theta}_1 \dot{\theta}_1^2}{2} \left[\frac{m_1}{l_1 \ddot{\theta}_1 + a_0} + \frac{k_1^3 m_2}{k_1 l_1 \ddot{\theta}_1 + a_0} + \frac{k_2^3 m_3}{k_2 l_1 \ddot{\theta}_1 + a_0} \right]. \tag{16}$$

The MOND Lagrangian L of the system has the form,

$$L = \frac{l_1^3 \ddot{\theta}_1 \dot{\theta}_1^2}{2} \left[\frac{m_1}{l_1 \ddot{\theta}_1 + a_0} + \frac{k_1^3 m_2}{k_1 l_1 \ddot{\theta}_1 + a_0} + \frac{k_2^3 m_3}{k_2 l_1 \ddot{\theta}_1 + a_0} \right] - V(\theta_1). \tag{17}$$

Solving Euler-Lagrange equation, we get

$$\frac{m_1 l_1^3 \ddot{\theta}_1^2}{(l_1 \ddot{\theta}_1 + a_0)} \left[1 - \frac{a_0}{l_1 \ddot{\theta}_1 + a_0} \right] + \frac{m_2 k_1^3 l_1^3 \ddot{\theta}_1^2}{(k_1 l_1 \ddot{\theta}_1 + a_0)} \left[1 - \frac{a_0}{k_1 l_1 \ddot{\theta}_1 + a_0} \right] + \frac{m_3 k_2^3 l_1^3 \ddot{\theta}_1^2}{(k_2 l_1 \ddot{\theta}_1 + a_0)} \left[1 - \frac{a_0}{k_2 l_1 \ddot{\theta}_1 + a_0} \right] = -\frac{\partial V}{\partial \theta_1}.$$

It's Newtonian and MOND analogue are as follows:

$$(m_1 + k_1^2 m_2 + k_2^2 m_3)\, l_1^2 \ddot{\theta}_1 \approx -\frac{\partial V}{\partial \theta_1}, \tag{18}$$

$$(m_1 + k_1^3 m_2 + k_2^3 m_3)\, l_1^3 \ddot{\theta}_1^2 / a_0 \approx -\frac{\partial V}{\partial \theta_1}. \tag{19}$$

Introducing a new potential $\frac{V}{l}$ instead of V, and restoring the same symbol V for MOND potential, above (18) and (19) assume the forms

$$(m_1 + k_1^2 m_2 + k_2^2 m_3)\, l_1 \ddot{\theta}_1 \approx -\frac{\partial V}{\partial \theta_1}, \tag{20}$$

and

$$(m_1 + k_1{}^3 m_2 + k_2{}^3 m_3)\, l_1^2 \ddot{\theta}_1{}^2 / a_0 \;\; \approx -\frac{\partial V}{\partial \theta_1}\,. \tag{21}$$

It seems that both Newtonian and MOND outcome have same sort of symmetry as is viewed from the pair of equations [(13), (20)] and [(14), (21)]. This enables us to rewrite the generalization to a system of n-pendulums in MOND and the same is summarized in the next section.

4 The System of n-Pendulums in MOND

Continuing the process detailed in the previous sections, we can generalize the equations to a system of n-pendulums as follows:

Newtonian analogue:

$$(m_1 + k_1{}^2 m_2 + k_2{}^2 m_3 + k_3{}^2 m_4 + ... + k_{n-1}{}^2 m_n)\, l_1 \ddot{\theta}_1 \approx -\frac{\partial V}{\partial \theta_1}\,. \tag{22}$$

MOND analogue:

$$(m_1 + k_1{}^3 m_2 + k_2{}^3 m_3 + k_3{}^3 m_4 + ... + k_{n-1}{}^3 m_n)\, l_1^2 \ddot{\theta}_1{}^2 / a_0 \;\; \approx -\frac{\partial V}{\partial \theta_1}\,. \tag{23}$$

The pattern of the above equations is worth noting.

5 Conclusion

The Newtonian analogue and the MOND analogue are contained in the equations (22) and (23) respectively.

Note

We can also get the same result by the method of mathematical induction.

Acknowledgements

We are thankful to Professor T M Karade for their fruitful suggestions.

Competing Interests

Authors have declared that no competing interests exist.

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APPENDIX

Derivation of (4)

$$v_1^2 = \dot{x}_1^2 + \dot{y}_1^2,\ v_2^2 = \dot{x}_2^2 + \dot{y}_2^2.$$

As $x_2 = k_1 x_1,\ y_2 = k_1 y_1 \ \Rightarrow v_2^2 = k_1^2 v_1^2$

$$\dot{v}_2 = k_1 \dot{v}_1$$

$$v_1 = l_1 \dot{\theta}_1 \Rightarrow \dot{v}_1 = l_1 \ddot{\theta}_1$$

$$\dot{v}_2 = k_1 l_1 \ddot{\theta}_1$$

$$T = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$

$$T = \frac{1}{2} m_1 l_1^2 \dot{\theta}_1^2 + \frac{1}{2} m_2 k_1^2 l_1^2 \dot{\theta}_1^2$$

$$T = \frac{1}{2} l_1^2 \dot{\theta}_1^2 \left[m_1 + m_2 k_1^2 \right]$$

Derivation of (5)

$$T = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$

$$T = \frac{1}{2} m_1 l_1^2 \dot{\theta}_1^2 \left[\frac{l_1 \ddot{\theta}_1}{l_1 \ddot{\theta}_1 + a_0} \right] + \frac{1}{2} m_2 k_1^2 l_1^2 \dot{\theta}_1^2 \left[\frac{k_1 l_1 \ddot{\theta}_1}{k_1 l_1 \ddot{\theta}_1 + a_0} \right]$$

$$T = \frac{l_1^3 \ddot{\theta}_1 \dot{\theta}_1^2}{2} \left[\frac{m_1}{l_1 \ddot{\theta}_1 + a_0} + \frac{k_1^3 m_2}{k_1 l_1 \ddot{\theta}_1 + a_0} \right]$$

Derivation of (15)

$$v_1^2 = \dot{x}_1^2 + \dot{y}_1^2,\ v_2^2 = \dot{x}_2^2 + \dot{y}_2^2,\ v_3^2 = \dot{x}_3^2 + \dot{y}_3^2$$

As $x_2 = k_1 x_1,\ y_2 = k_1 y_1,\ and\ x_3 = k_2 x_1,\ y_3 = k_2 y_1,$

$$v_2^2 = k_1^2 v_1^2,\ v_3^2 = k_2^2 v_1^2$$

$$v_1 = l_1\dot{\theta}_1 \Rightarrow \dot{v}_1 = l_1\ddot{\theta}_1 \;\; and \; v_2 = k_1 l_1\dot{\theta}_1 \Rightarrow \dot{v}_2 = k_1 l_1\ddot{\theta}_1 \;\; also \; v_3 = k_2 l_1\dot{\theta}_1 \Rightarrow \dot{v}_3 = k_2 l_1\ddot{\theta}_1$$

$$T = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 + \frac{1}{2}m_3 v_3^2$$

$$T = \frac{1}{2}m_1 l_1^2\dot{\theta}_1^2 + \frac{1}{2}m_2 k_1^2 l_1^2\dot{\theta}_1^2 + \frac{1}{2}m_3 k_2^2 l_1^2\dot{\theta}_1^2$$

$$T = \frac{1}{2}l_1^2\dot{\theta}_1^2\left[m_1 + m_2 k_1^2 + m_3 k_2^2\right]$$

Derivation of (16)

$$T = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 + \frac{1}{2}m_3 v_3^2$$

$$T = \frac{1}{2}m_1 l_1^2\dot{\theta}_1^2\left[\frac{l_1\ddot{\theta}_1}{l_1\ddot{\theta}_1 + a_0}\right] + \frac{1}{2}m_2 k_1^2 l_1^2\dot{\theta}_1^2\left[\frac{k_1 l_1\ddot{\theta}_1}{k_1 l_1\ddot{\theta}_1 + a_0}\right] + \frac{1}{2}m_3 k_2^2 l_1^2\dot{\theta}_1^2\left[\frac{k_2 l_1\ddot{\theta}_1}{k_2 l_1\ddot{\theta}_1 + a_0}\right]$$

$$T = \frac{l_1^3\ddot{\theta}_1\dot{\theta}_1^2}{2}\left[\frac{m_1}{l_1\ddot{\theta}_1 + a_0} + \frac{k_1^3 m_2}{k_1 l_1\ddot{\theta}_1 + a_0} + \frac{k_2^3 m_3}{k_2 l_1\ddot{\theta}_1 + a_0}\right]$$

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