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# IGPR-Continuity and Compactness in Intuitionistic Topological Space

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## Abstract

In this paper, the notion of intuitionistic generalised pre-regular continuity is introduced. Also intuitionistic generalised pre-regular compactness is defined in intuitionistic topological spaces and several preservation properties are obtained. Moreover, some properties of intuitionistic generalised pre-regular continuity are outlined.

*Keywords: Intuitionistic set; intuitionistic topology; intuitionistic cover; Igpr-continuous; Igpr compact.*

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## 1 Introduction

After the introduction of the concept of fuzzy set by Zadeh [1], Atanassov[2] proposed the concept of intuitionistic fuzzy sets. Coker [3] introduced the concept of intuitionistic sets and intuitionistic points in 1996 and intuitionistic fuzzy topological spaces[4] in 1997. Also in 2000,Coker [5] developed the concept of intuitionistic topological spaces with intuitionistic sets and investigated basic properties of continuous functions and compactness. Selma ozcag and Dogan Coker [6,7] also examined

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connectedness in intuitionistic topological spaces. Later, several researchers [8] studied some weak forms of intuitionistic topological spaces. In this paper, we study some properties of intuitionistic generalised pre-regular continuity and intuitionistic generalised pre-regular continuity compactness in intuitionistic topological spaces.

## 1.1 Preliminaries

We recall some definitions and results which are useful for this sequel. Throughout the present study, a space X means an intuitionistic topological space $(X, \tau)$ and Y means an intuitionistic topological space $(Y, \sigma)$ unless otherwise mentioned.

**Definition 1.1.** [3] Let X be a nonempty set. An intuitionistic set (IS for short) A is an object having the form A $= < X, A_1, A_2 >$, where $A_1$ and $A_2$ are subsets of X satisfying $A_1 \cap A_2 = \phi$. The set $A_1$ is called the set of members of A, while $A_2$ is called the set of non-members of A.

**Definition 1.2.** [3] Let X be a nonempty set and let A,B be intuitionistic sets in the form A $= < X, A_1, A_2 >$, B $= < X, B_1, B_2 >$ respectively. Then

(a) A $\subseteq$ B iff $A_1 \subseteq B_1$ and $A_2 \supseteq B_2$.

(b) A = B iff A $\subseteq$ B and B $\subseteq$ A.

(c) $\overline{A} = < X, A_2, A_1 >$.

(d) A - B $= A \cap \overline{B}$.

(e) $\underset{\sim}{\phi} = < X, \phi, X >$, $\underset{\sim}{X} = < X, X, \phi >$.

(f) A $\cup$ B $= < X, A_1 \cup B_1, A_2 \cap B_2 >$.

(g) A $\cap$ B $= < X, A_1 \cap B_1, A_2 \cup B_2 >$.

(h) []A $= < X, A_1, (A_1)^c >$.

(i) <>A $= < X, (A_2)^c, A_2 >$.

Furthermore, let $\{A_i : i \in J\}$ be an arbitrary family of intuitionistic sets in X, where $A_i = < X, A_i^{(1)}, A_i^{(2)} >$. Then

(j) $\bigcap A_i = < X, \cap A_i^{(1)}, \cup A_i^{(2)} >$.

(k) $\bigcup A_i = < X, \cup A_i^{(1)}, \cap A_i^{(2)} >$.

**Definition 1.3.** [4, 5] An intuitionistic topology ( IT for short ) on a nonempty set X is a family $\tau$ of IS's in X containing $\underset{\sim}{\phi}$, $\underset{\sim}{X}$ and closed under finite infima and arbitrary suprema. The pair $(X, \tau)$ is called an intuitionistic topological space $(ITS\ for\ short)$. Any intuitionistic set in $\tau$ is known as an intuitionistic open set $(IOS\ for\ short)$ in X and the complement of IOS is called intuitionistic closed set $(ICS\ for\ short)$.

**Proposition 1.1.** [5] *Let $(X, \tau)$ be an ITS on X and $A = < X, A_1, A_2 >$ be an IS in X. Then the several intuitionistic topologies $[(a), (b)]$ and general topologies $[(c), (d)]$ generated by $(X, \tau)$ are*

*(a)* $\tau_{0,1} = \{[]A : A \in \tau\}$

*(b)* $\tau_{0,2} = \{<> A : A \in \tau\}$

*(c)* $\tau_1 = \{A_1 :< X, A_1, A_2 >\in \tau\}$

*(d)* $\tau_2 = \{(A_2)^c :< X, A_1, A_2 >\in \tau\}$

**Definition 1.4.** [3] Let $(X, \tau)$ be an ITS and A $= < X, A_1, A_2 >$ be an IS in X. Then the closure and interior of an intuitionistic set A is defined as
Icl(A)$= \bigcap$ {K : K is an ICS in X and A $\subseteq$ K}
Iint(A) $= \bigcup$ {G : G is an IOS in X and G $\subseteq$ A}

It can be shown that Icl(A) is an ICS and Iint(A) is an IOS in X and A is an ICS in X iff Icl(A)= A and is an IOS in X iff Iint(A) = A.

**Definition 1.5.** [3] Let X be a nonempty set and a $\in$ X. Then the IS $\underset{\sim}{a}$ defined by $\underset{\sim}{a} = < X, \{a\}, \{a\}^c >$ is called an intuitionistic point ($IP$ *for short*) in X. The intuitionistic point $\underset{\sim}{a}$ is said to be contained in A $= < X, A_1, A_2 >$ ( i.e $\underset{\sim}{a} \in$ A) if and only if a $\in A_1$.

**Definition 1.6.** [9] Let $(X, \tau)$ be an ITS. An intuitionistic set A of X is said to be intuitionistic regular open (intuitionistic regular closed ) if A $= Iint(Icl(A))$ $(A = Icl(Iint(A)))$.

**Definition 1.7.** [9] Let $(X, \tau)$ be an ITS and let A $= < X, A_1, A_2 >$ be an intuitionistic set. Then A is said to be intuitionistic generalized pre-regular closed (*Igpr-closed*) if Ipcl$(A) \subseteq$ U whenever A $\subseteq$ U and U is intuitionistic regular open in X. The class of all Igpr-closed subsets of $(X, \tau)$ is denoted by IGPRC$(\tau)$.

The complement of intuitionistic generalized pre-regular closed sets are intuitionistic generalized pre-regular open (Igpr-open) and the class of all Igpr-open subsets of $(X, \tau)$ is denoted by IGPRO$(\tau)$.

**Definition 1.8.** [4] Let $(X, \tau)$ be a nonempty ITS and let A $= < X, A_1, A_2 >$ be IS. Then A is said to be

(i) intuitionistic regular generalized closed (*Irg-closed*) if Icl(A) $\subseteq$ U whenever A $\subseteq$ U and U is intuitionistic regular open in X.
(ii) intuitionistic generalized $\alpha$ closed (*Ig$\alpha$-closed*) if $I\alpha cl(A) \subseteq$ U whenever A $\subseteq$ U and U is intuitionistic open in X.

**Definition 1.9.** [3] Let X,Y be two nonempty sets and f :X $\rightarrow$ Y be a function. $(a)$ If B $= < X, B_1, B_2 >$ is an IS in Y, then the preimage of B under f, denoted by $f^{-1}(B)$,is the IS in X defined by $f^{-1}(B) = < X, f^{-1}(B_1), f^{-1}(B_2) >$.
$(b)$ If A $= < X, A_1, A_2 >$ is an IS in X, then the image of A under f, denoted by $f(A)$, is the IS in Y defined by $f(B) = < Y, f(A_1), f(A_2) >$, where $f(A_2) = (f((A_2)^c))^c$.

**Definition 1.10.** [3] Let $(X, \tau)$ and $(Y, \sigma)$ be two intuitionistic topological spaces and f :X $\rightarrow$ Y be a function.Then f is said to be continuous iff the preimage of each ICS in Y is intuitionistic closed in X.

**Corollary 1.1.** [4, 5] *Let A, $A_i$ $(i \in J)$ be IS's in X, B, $B_j$ $(j \in K)$ be IS's in Y and f : $(X, \tau) \rightarrow (Y, \sigma)$ be a function. Then*

$(a)$ $A_1 \subseteq A_2 \Rightarrow f(A_1) \subseteq f(A_2)$, $B_1 \subseteq B_2 \Rightarrow f^{-1}(B_1) \subseteq f^{-1}(B_2)$
$(b)$ $A \subseteq f^{-1}(f(A))$ *and if f is injective, then* $A = f^{-1}(f(A))$
$(c)$ $f(f^{-1}(B)) \subseteq B$ *and if f is surjective then* $f(f^{-1}(B)) = B$
$(d)$ $f(\cup A_i) = \cup f(A_i)$; $f(\cap A_i) \subseteq \cap f(A_i)$ *and if f is injective then* $f(\cap A_i) = \cap f(A_i)$
$(e)$ $f^{-1}(\cup B_i) = \cup f^{-1}(B_i)$; $f^{-1}(\cap B_i) \subseteq \cap f^{-1}(B_i)$
$(f)$ *If f is surjective then* $\overline{f(A)} \subseteq f(\overline{A})$. *Further if f is injective then* $\overline{f(A)} = f(\overline{A})$
$(g)$ $f^{-1}(\overline{B}) = \overline{f^{-1}(B)}$

**Proposition 1.2.** [5] *The following statements are equivalent :*

(a) $f :(X, \tau) \to (Y, \sigma)$ *is continuous.*
(b) $f^{-1}(int(B)) \subseteq int(f^{-1}(B))$ *for each IS B in Y*
(c) $cl(f^{-1}(int(B))) \subseteq f^{-1}(cl(B))$ *for each IS B in Y.*

**Definition 1.11.** [6] If there exists an intuitionistic regular open set A in X such that $\underset{\sim}{\phi} \neq A \neq \underset{\sim}{X}$, then X is called super disconnected. X is called super connected, if X is not super disconnected.

# 2 Igpr-Continuity In Intuitionistic Topological Spaces

**Definition 2.1.** A mapping f : $(X, \tau) \longrightarrow (Y, \sigma)$ is called intuitionistic generalised pre-regular continuous (*briefly Igpr*-continuous) if the preimage of every intuitionistic closed set of Y is Igpr-closed in X. i.e., $f^{-1}(V)$ is Igpr-closed in $(X, \tau)$ for every intuitionistic closed set V of $(Y, \sigma)$ .

**Theorem 2.1.** *A mapping f : $(X, \tau) \longrightarrow (Y, \sigma)$ is Igpr-continuous iff the preimage of every intuitionistic open set of Y is Igpr-open in X.*

*Proof.* Let B = $< Y, B_1, B_2 >$ be an intuitionistic open set of Y. Then $f^{-1}(B^c)= f^{-1}(< Y, B_2, B_1 >) = < X, f^{-1}(B_2), f^{-1}(B_1) >$. Also $(f^{-1}(B))^c = (f^{-1}(< Y, B_1, B_2 >)^c = (< X, f^{-1}(B_1), f^{-1}(B_2) >)^c = < X, f^{-1}(B_2), f^{-1}(B_1) >$. Since $f^{-1}(B^c) = (f^{-1}(B))^c$, for every intuitionistic set B of Y,$f^{-1}(B^c)$ is Igpr-closed in Y. So, $f^{-1}(B)$ is Igpr-open in X. Hence f is Igpr-continuous. The converse part follows from the definition. □

**Proposition 2.1.** *Let $(X, \tau)$ and $(Y, \sigma)$ be two intuitionistic topological spaces. If f : $(X, \tau_{0,1}) \longrightarrow (Y, \sigma_{0,1})$ and f : $(X, \tau_{0,2}) \longrightarrow (Y, \sigma_{0,2})$ are Igpr-continuous, then f : $(X, \tau) \longrightarrow (Y, \sigma)$ is Igpr-continuous.*

*Proof.* Let B = $< Y, B_1, B_2 >$ be an intuitionistic open set of Y. By hypothesis, f : $(X, \tau_{0,1}) \longrightarrow (Y, \sigma_{0,1})$ and f : $(X, \tau_{0,2}) \longrightarrow (Y, \sigma_{0,2})$ are Igpr-continuous. So there exist an Igpr-open sets $f^{-1}(< Y, B_1, (B_1)^c >) = < X, f^{-1}(B_1), f^{-1}(B_1)^c >$ in $(X, \tau_{0,1})$ and $f^{-1}(< Y, (B_2)^c, B_2 >) = < X, f^{-1}(B_2)^c, f^{-1}(B_2) >$ in $(X, \tau_{0,2})$. Since $B_2 \subset (B_1)^c$ and $B_1 \subset (B_2)^c$, $< X, f^{-1}(B_1), f^{-1}(B_2) > \subseteq < X, f^{-1}(B_1), f^{-1}(B_1)^c >$ or $< X, f^{-1}(B_2)^c, f^{-1}(B_2) >$. Hence $< X, f^{-1}(B_1), f^{-1}(B_2) >$ is Igpr-open in X and so f : $(X, \tau) \longrightarrow (Y, \sigma)$ is Igpr-continuous. □

**Proposition 2.2.** *Let $(X, \tau)$ and $(Y, \sigma)$ be two intuitionistic topological spaces. If f : $(X, \tau_1) \longrightarrow (Y, \sigma_1)$ and f : $(X, \tau_2) \longrightarrow (Y, \sigma_2)$ are gpr-continuous, then f : $(X, \tau) \longrightarrow (Y, \sigma)$ is Igpr-continuous.*

*Proof.* Similar to Proposition 2.1. □

**Theorem 2.2.** *Let f : $(X, \tau) \longrightarrow (Y, \sigma)$ be intuitionistic continuous. Then f is Igpr-continuous but not conversely.*

*Proof.* Let f : $(X, \tau) \longrightarrow (Y, \sigma)$ be intuitionistic continuous and A be any intuitionistic closed set in Y. Then the inverse image $f^{-1}(A)$ is intuitionistic closed in X. Since every intuitionistic closed set is Igpr-closed,$f^{-1}(A)$ is Igpr-closed in X. So f is Igpr-continuous. □

*Remark* 2.1. The converse of the above Theorem 2.2 is not true as seen from the following Example.

**Example 2.3.** *Let* $X=\{a,b,c\}$ *with intuitionistic topology* $\tau=\{\underset{\sim}{\phi},<X,\{a\},\{b\}>,<X,\{b\},\{c\}>,<X,\{a,b\},\phi>,<X,\phi,\{b,c\}>,\underset{\sim}{X}\}$ *and* $Y=\{1,2,3\}$ *with intuitionistic topology* $\sigma=\{\underset{\sim}{\phi},<Y,\{1\},\{2,3\}>,<Y,\{2,3\},\phi>,<Y,\phi,\{2\}>,\underset{\sim}{Y}\}$ *. Define* $f:(X,\tau)\longrightarrow(Y,\sigma)$ *by* $f(a)=1$*,* $f(b)=2$ *and* $f(c)=3$*. Then* $f:(X,\tau)\longrightarrow(Y,\sigma)$ *is Igpr-continuous but not intuitionistic continuous. Also, f:* $(X,\tau_1)\longrightarrow(Y,\sigma_1)$*,* $f:(X,\tau_2)\longrightarrow(Y,\sigma_2)$*,* $f:(X,\tau_{0,1})\longrightarrow(Y,\sigma_{0,1})$*,* $f:(X,\tau_{0,2})\longrightarrow(Y,\sigma_{0,2})$ *are Igpr-continuous but* $f:(X,\tau_1)\longrightarrow(Y,\sigma_1)$*,* $f:(X,\tau_{0,1})\longrightarrow(Y,\sigma_{0,1})$ *are not intuitionistic continuous.*

**Proposition 2.3.** *Let* $f:(X,\tau)\longrightarrow(Y,\sigma)$ *be Irg-continuous. Then f is Igpr-continuous.*

*Proof.* Let B be intuitionistic closed in $(Y,\sigma)$. Since f is Irg-continuous, $f^{-1}(B)$ is Irg closed in $(X,\tau)$. By the proposition 3.4 of [7], $f^{-1}(B)$ is Igpr-closed. Hence f is Igpr-continuous. □

*Remark* 2.2. The converse of the above proposition does not need to be true as seen from the following example.

**Example 2.4.** *Let* $X=\{a,b,c\}$ *with intuitionistic topology* $\tau=\{\underset{\sim}{\phi},\underset{\sim}{X},A,B,C\}$ *where* $A=<X,\{c\},\{a,b\}>$*,* $B=<X,\{a\},\{b,c\}>$ *and* $C=<X,\{a,c\},\{b\}>$ *and let* $Y=\{1,2,3\}$ *with intuitionistic topology* $\sigma=\{\underset{\sim}{\phi},P,Q,R,S,\underset{\sim}{Y}\}$ *where* $P=<Y,\{1\},\{2\}>$*,* $Q=<Y,\{2\},\{3\}>$*,* $R=<Y,\phi,\{2,3\}>$*,* $S=<Y,\{1,2\},\phi>$*. Define* $f:(X,\tau)\longrightarrow(Y,\sigma)$ *by* $f(a)=1$*,* $f(b)=3$ *and* $f(c)=2$*. Then f is Igpr-continuous but not Irg-continuous.*

**Proposition 2.4.** *Let* $f:(X,\tau)\longrightarrow(Y,\sigma)$ *be Iαg-continuous. Then f is Igpr-continuous.*

*Proof.* Let B be intuitionistic closed in $(Y,\sigma)$. Since f is Iαg-continuous, $f^{-1}(B)$ is Iαg-closed in $(X,\tau)$. Since every closed set is Igpr-closed, $f^{-1}(B)$ is Igpr closed. Hence f is Igpr-continuous. □

*Remark* 2.3. The converse of the above proposition does not need to be true as seen from the following example.

**Example 2.5.** *Let* $X=\{a,b,c\}$ *and* $\tau=\{\underset{\sim}{\phi},A,B,C,D,E,F,\underset{\sim}{X}\}$ *where* $A=<X,\phi,\{a,b\}>$*,* $B=<X,\{c\},\{a,b\}>$*,* $C=<X,\phi,\{b,c\}>$*,* $D=<X,\{c\},\{b\}>$*,* $E=<X,\{a,c\},\{b\}>$ *and* $F=<X,\phi,\{b\}>$ *and let* $Y=\{1,2,3\}$ *with intuitionistic topology* $\sigma=\{\underset{\sim}{\phi},P,Q,R,S,\underset{\sim}{Y}\}$ *where* $P=<Y,\{1,2\},\{3\}>$*,* $Q=<Y,\{2,3\},\{1\}>$ *and* $R=<Y,\{1,3\},\{2\}>$*. Define* $f:(X,\tau)\longrightarrow(Y,\sigma)$ *by* $f(a)=1$*,* $f(b)=2$ *and* $f(c)=3$*. Then f is Igpr-continuous but not Iαg-continuous.*

**Proposition 2.5.** *A mapping* $f:(X,\tau)\longrightarrow(Y,\sigma)$ *is Igpr-continuous if* $(X,\tau)$ *is intuitionistic super-connected.*

*Proof.* If $(X,\tau)$ is intuitionistic super-connected, then the only intuitionistic regular open subsets of $(X,\tau)$ are $\underset{\sim}{\phi}$ and $\underset{\sim}{X}$ which implies all the subsets of X are Igpr-closed. Hence the preimage of every intuitionistic closed set of Y is Igpr-closed in X. □

**Theorem 2.6.** *Let* $f:(X,\tau)\longrightarrow(Y,\sigma)$ *be Igpr-continuous. Then* $f(Igprcl(A))\subset Icl(f(A))$ *for every intuitionistic subset A of X.*

*Proof.* Let A be an intuitionistic subset of X. Then $Icl(f(A))$ is intuitionistic closed in $(Y,\sigma)$. Since f is Igpr-continous , $f^{-1}(Icl(f(A)))$ is Igpr-closed in X. And $A\subset f^{-1}(f(A))\subset f^{-1}(Icl(f(A)))$ implies $Igprcl(A)\subset Igprcl(f^{-1}(Icl(f(A))))\ f^{-1}(Icl(f(A)))$. Hence $f(Igprcl(A))\subset Icl(f(A))$. □

**Definition 2.2.** Let $(X,\tau)$ be an intuitionistic topological space then $\tau^*=$
$\{A\subset X/Igprcl(\underset{\sim}{X}-A)=\underset{\sim}{X}-A\}$

**Theorem 2.7.** *Every Igpr-closed set is intuitionistic closed iff $\tau^* = \tau$ holds.*

*Proof.* Let A $\in \tau^*$. Then $Igprcl(\underset{\sim}{X} - A) = \underset{\sim}{X} - A$. Since $\tau^* = \tau$ every Igpr-closed set is intuitionistic closed. The other part can be proved easily. □

*Remark* 2.4. If $\tau^* = \tau$ in the intuitionistic topological space $(X, \tau)$, then intuitionistic continuity and Igpr-continuity coincide.

*Proof.* Let f: $(X, \tau) \longrightarrow (Y, \sigma)$ be Igpr-continuous. Then $f^{-1}(A)$ is Igpr-closed for every intuitionistic closed set A of Y. Since $\tau^* = \tau$ and by Theorem 2.7, $f^{-1}(A)$ is intuitionistic closed. Hence f is intuitionistic continuous. □

**Theorem 2.8.** *If IGPRO$(X)$ forms a topology in intuitionistic space then $\tau^*$ is also a topology.*

*Proof.* Obvious □

**Theorem 2.9.** *Consider the mapping $f : (X, \tau^*) \longrightarrow (Y, \sigma)$. Then the following statements are equivalent.*

*$(i)$ For every intuitionistic subset A of X, $f(Igprcl(A)) \subset Icl(f(A))$. $(ii)$ If $\tau^*$ is a topology, then $f : (X, \tau^*) \longrightarrow (Y, \sigma)$ is intuitionistic continuous.*

*Proof.* $(i) \Longrightarrow (ii)$ : Let A be intuitionistic closed in $(Y, \sigma)$.
From $(i)$,$f(Igprcl(f^{-1}(A))) \subset Icl(f(f^{-1}(A))) \subset Icl(A) =$ A. So $Igprcl(f^{-1}(A)) \subset f^{-1}(A)$. Also $f^{-1}(A) \subset Igprcl(f^{-1}(A))$. Thus $(f^{-1}(A))^c \in \tau^*$ which implies $f^{-1}(A)$ is intuitionistic closed in $(X, \tau^*)$ . So f is intuitionistic continuous. $(ii) \Longrightarrow (i)$ : For every subset A of X, $Icl(f(A))$ is intuitionistic closed in $(Y, \sigma)$. Then from $(ii)$, $f^{-1}(Icl(f(A)))$ is intuitionistic closed in $\tau^*$. So, $Igprcl(f^{-1}(Icl(f(A)))) = f^{-1}(Icl(f(A)))$ which implies $f(Igprcl(f^{-1}(cl(f(A))))) \subset Icl(f(A))$. Since f is intuitionistic continuous, A $\subset Icl(A) \subset Icl(f^{-1}(f(A))) \subset f^{-1}(Icl(f(A)))$. Hence
$f(Igprcl(A)) \subset f(Igprcl(f^{-1}(cl(f(A))))) \subset Icl(f(A))$. □

# 3 Igpr-Compactness In Intuitionistic Topological Spaces

**Definition 3.1.** Let $(X, \tau)$ be an intuitionistic topological space. If a family $\{< X, {G_i}^{(1)}, {G_i}^{(2)} >; i \in \Lambda\}$ of Igpr-open sets in X satisfies the condition $\cup \{< X, {G_i}^{(1)}, {G_i}^{(2)} >; i \in \Lambda\} = \underset{\sim}{X}$ , then it is called an Igpr-open cover of X.

A finite subfamily of an Igpr-open cover $\{< X, {G_i}^{(1)}, {G_i}^{(2)} >; i \in J\}$ of X,which is also an Igpr-open cover of X is called a finite subcover of $\{< X, {G_i}^{(1)}, {G_i}^{(2)} >; i \in \Lambda\}$.

**Definition 3.2.** An intuitionistic topological space $(X, \tau)$ is said to be Igpr-compact iff each Igpr-open cover has a finite subcover.

**Definition 3.3.** Let $(X, \tau)$ be an intuitionistic topological space and A be an IS in X. The family $\{< X, {G_i}^{(1)}, {G_i}^{(2)} >; i \in \Lambda\}$ of Igpr-open sets in X is called a Igpr-open cover of A if A $\subseteq \cup\{< X, {G_i}^{(1)}, {G_i}^{(2)} >; i \in \Lambda\}$.

**Definition 3.4.** An IS A $= < X, A^{(1)}, A^{(2)} >$ in an ITS $(X, \tau)$ is called Igpr-compact iff every Igpr-open cover of A has a finite sub cover. Also we can define an IS A $= < X, A^{(1)}, A^{(2)} >$ in $(X, \tau)$ is Igpr-compact iff for each family $\mathcal{G} = \{$G$_i : i \in \Lambda\}$ where G$_i = \{< X, G_i^{(1)}, G_i^{(2)} >: i \in \Lambda\}$ of Igpr-open sets in X, A$^{(1)} \subseteq \cup_{i\in\Lambda} G_i^{(1)}$ and A$^{(2)} \supseteq \cup_{i\in\Lambda} G_i^{(2)}$, there exists a finite subfamily $\{$G$_i : i = 1, 2, ..., n\}$ of $\mathcal{G}$ such that A$^{(1)} \subseteq \cup_{i=1}^{n} G_i^{(1)}$ and A$^{(2)} \supseteq \cup_{i=1}^{n} G_i^{(2)}$.

**Proposition 3.1.** *Let $(X, \tau)$ be an intuitionistic topological space. Then $(X, \tau)$ is Igpr-compact iff the ITS $(X, \tau_{0,1})$ is Igpr-compact.*

*Proof.* Neccesity: Let $(X, \tau)$ be Igpr-compact and consider an Igpr-open cover $\{[]G_j : j \in \Lambda\}$ of X in $(X, \tau_{0,1})$. Since $\cup([]G_j) = \underset{\sim}{X}$, we obtain $\cup G_j^{(1)}$ = X and hence $G_j^{(2)} \subseteq (G_j^{(1)})^c \Rightarrow \cap G_j^{(2)} \subseteq (\cup G_j^{(1)})^c$ $= \phi \Rightarrow \cup G_j = \underset{\sim}{X}$. Since $(X, \tau)$ is Igpr-compact,there exists $G_1, G_2, ......, G_n$ such that $\overset{n}{\underset{i=1}{\cup}} G_i = \underset{\sim}{X}$ which implies $\overset{n}{\underset{i=1}{\cup}} G_i^{(1)}$ = X and $\overset{n}{\underset{i=1}{\cap}} G_i^{(2)} = \phi$. So $(X, \tau_{0,1})$ is Igpr-compact.
Sufficiency: Suppose $(X, \tau_{0,1})$ is Igpr-compact. Consider an Igpr-open cover $\{G_j : j \in \Lambda\}$ of X in $(X, \tau)$. Since $\cup(G_j) = \underset{\sim}{X}$, we obtain $\cup G_j^{(1)}$ = X and hence $\cap(G_j^{(1)})^c = \phi \Rightarrow \cap G_j^{(2)} \subseteq (\cup G_j^{(1)})^c = \phi \Rightarrow$ $\cup G_j = \underset{\sim}{X}$. Since $(X, \tau_{0,1})$ is Igpr-compact,there exists $G_1, G_2, ......, G_n$ such that $\overset{n}{\underset{i=1}{\cup}} []G_i = \underset{\sim}{X}$ which implies $\overset{n}{\underset{i=1}{\cup}} G_i^{(1)}$ = X and $\overset{n}{\underset{i=1}{\cap}} (G_i^{(1)})^c = \phi$. Hence $G_i^{(1)} \subseteq (G_i^{(2)})^c \Rightarrow X = \overset{n}{\underset{i=1}{\cup}} G_i^{(1)} \subseteq \overset{n}{\underset{i=1}{\cap}} (G_i^{(2)})^c \Rightarrow \overset{n}{\underset{i=1}{\cap}} G_i^{(2)}$ $= \phi$. Thus $\overset{n}{\underset{i=1}{\cup}} G_i = \underset{\sim}{X}$. So $(X, \tau)$ is Igpr-compact. □

**Proposition 3.2.** *The ITS $(X, \tau)$ is Igpr-compact iff $(X, \tau_1)$ is Igpr-compact.*

*Proof.* Similar to Proposition 3.1. □

**Proposition 3.3.** *Let $f : (X, \tau) \longrightarrow (Y, \sigma)$ be a surjective Igpr-continuous mapping. If $(X, \tau)$ is Igpr-compact then $(Y, \sigma)$ is intuitionistic compact.*

*Proof.* Let $\{< X, G_i^{(1)}, G_i^{(2)} >: i \in \Lambda\}$ be any intuitionistic open cover of Y. Since f is Igpr-continuous, $\{f^{-1}(G_i) : i \in \Lambda\}$ is an Igpr-open cover of X. Since $(X, \tau)$ is Igpr-compact, it has a finite subcover $\{f^{-1}(G_1), f^{-1}(G_2), f^{-1}(G_3), ...., f^{-1}(G_n)\}$ such that $\overset{n}{\underset{i=1}{\cup}} f^{-1}(G_i^{(1)}) = \underset{\sim}{X}$ and $\overset{n}{\underset{i=1}{\cap}} f^{-1}(G_i^{(2)})$ $= \underset{\sim}{\phi}$. i.e, $f^{-1}(\overset{n}{\underset{i=1}{\cup}} G_i^{(1)})$ = X and $f^{-1}(\overset{n}{\underset{i=1}{\cap}} G_i^{(2)}) = \phi \Rightarrow \overset{n}{\underset{i=1}{\cup}} G_i^{(1)} = f(X)$ and $\overset{n}{\underset{i=1}{\cap}} G_i^2 = f(\phi)$. Since f is surjective $\{G_1, G_2, ....., G_n\}$ is an open cover of Y and hence $(Y, \sigma)$ is intuitionistic compact. □

**Corollary 3.1.** *Let $f : (X, \tau) \longrightarrow (Y, \sigma)$ be Igpr-continuous. If A is Igpr-compact in $(X, \tau)$, then $f(A)$ is intuitionistic compact in $(Y, \sigma)$.*

**Definition 3.5.** [10] Let $(X, \tau)$ and $(Y, \sigma)$ be two intuitionistic topological spaces and f : X → Y be a function. Then f is said to be Igpr-irresolute if the preimage of every Igpr-closed set of Y is Igpr-closed in X.

**Proposition 3.4.** *Let $f : (X, \tau) \longrightarrow (Y, \sigma)$ be an Igpr-irresolute mapping and if A is Igpr-compact relative to X, then $f(A)$ is Igpr-compact relative to Y.*

*Proof.* Let $\{G_i : i \in \Lambda\}$ be an Igpr-open set of Y such that $f(A) \subseteq \cup \{G_i : i \in \Lambda\}$. Then A $\subset \cup$ $\{f^{-1}(G_i) : i \in \Lambda\}$ where $f^{-1}(G_i)$ is Igpr-open in X for each i. Since A is Igpr-compact relative to X, there exists a finite sub collection $\{G_1, G_2, ....., G_n\}$ such that A$\subset \cup \{f^{-1}(G_i) : i = 1, 2, ..., n\}$. i.e., $f(A) \subset \cup \{G_i : i = 1, 2, ..., n\}$. Hence $f(A)$ is Igpr-compact relative to Y. □

**Proposition 3.5.** *Let $f : (X, \tau) \longrightarrow (Y, \sigma)$ be an Igpr-irresolute mapping.If X is Igpr-compact, then Y is also an Igpr-compact space .*

*Proof.* Let f : $(X, \tau) \longrightarrow (Y, \sigma)$ be an Igpr-irresolute mapping from Igpr-compact space $(X, \tau)$ onto an intuitionistic topological space $(Y, \sigma)$. Let $\{G_i : i \in \Lambda\}$ be an Igpr-open cover of Y. Then $\{f^{-1}(G_i) : i \in \Lambda\}$ is an Igpr-open cover of X. Since X is Igpr-compact, there is a finite subfamily $\{f^{-1}(A_{i_1}), f^{-1}(A_{i_2}), f^{-1}(A_{i_3}), ......, f^{-1}(A_{i_n})\}$ of $\{f^{-1}(A_i) : i \in \Lambda\}$ such that $\overset{n}{\underset{j=1}{\cup}} G_{i_j} = \underset{\sim}{X}$. Since f is onto, $f(\underset{\sim}{X}) = \underset{\sim}{X}$ and $f(\overset{n}{\underset{j=1}{\cup}} f^{-1}(G_{i_j})) = \overset{n}{\underset{j=1}{\cup}} f(f^{-1}(G_{i_j}))= \overset{n}{\underset{j=1}{\cup}} G_{i_j}$. It follows that $\overset{n}{\underset{j=1}{\cup}} G_{i_j} = \underset{\sim}{X}$ and the family $\{G_{i_1}, G_{i_2}, ..., G_{i_n}\}$ is an intuitonistic finite subcover of $\{G_i : i \in \Lambda\}$. Hence $(Y, \sigma)$ is an Igpr- compact. □

# 4 Conclusions

In this paper, the development of intuitionistic generalised pre-regular continuity and its various algebraic features in intuitionistic topological spaces are studied. Also intuitionistic generalised pre- regular compactness in intuitionistic topological spaces are also discussed and it is proved that $(X, \tau)$ is Igpr-compact if and only if $(X, \tau_{0,1})$ is Igpr-compact.

# Competing Interests

The authors declare that no competing interests exist.

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