British Journal of Mathematics & Computer Science
4(24): 3427-3443, 2014
ISSN: 2231-0851

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Price Dynamics of Maize in Ghana: An Application of Continuous –Time Delay Differential Equations

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Article Information

DOI: 10.9734/BJMCS/2014/13484
Editor(s):
(1) Xiaodi Li, School of Mathematical Sciences, Shandong Normal University Ji'nan, 250014, Shandong, P. R. China.
Reviewers:
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(4) Anonymous, Gazi University, Turkey.
Complete Peer review History: http://www.sciencedomain.org/review-history.php?iid=699&id=6&aid=6361

Received: 19 August 2014
Accepted: 20 September 2014
Published: 06 October 2014

Original Research Article

Abstract

This paper seeks to study the price dynamics of maize in Ghana, in the context of mathematical modelling using continuous-time cobweb models derived from linear and nonlinear delay differential equations. The stability conditions of two cobweb models: continuous-time linear and nonlinear models are discussed. The data is obtained from the Ministry of Food and Agriculture, Statistical Directorate Kumasi-Ghana, from 1994 to 2013.

The models performed on the assumptions that, maize has no equal substitutes and there are no exogenous shocks needed to generate price fluctuations so that market price would be determined by only the available supply in a single market.

From the results of the analysis performed on the real economic price and production data using numerical approach, the nonlinear delay differential model (formulated from linear demand and quadratic supply functions) showed oscillations between and around two equilibrium points and would neither converge. This result seems realistic as it appears to reflect real market conditions since an equilibrium price would not be compatible with the prevailing situation of high inflation, food insufficiency and/or producers' sensitivity towards price.

The linear model (formulated from linear demand and linear supply functions) on the other hand showed little oscillations and then converged towards an equilibrium point (zero

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equilibrium price). However, this seems quite unrealistic.
Effects of delay parameter τ on oscillations are also discussed. It is observed that oscillations (price fluctuations) are suppressed for $\tau \leq 0.5$, using the nonlinear delay equation. This is an indication that price fluctuations are reduced, if and only if, factors affected by time lag, such as the time necessary for increasing supply, buying new inputs, hiring workers or transporting commodities to market centres, building warehouse and reducing effects of natural constraints on crop yields are improved. On the contrary, the linear delay differential equation, for $\tau > 0.5$ or $\tau \leq 0.5$, would still be in stable equilibrium.
We draw inferences from this study that researchers should rather use nonlinear models instead of linear models in solving most real-life economic problems to avoid misleading conclusions.
Keywords: Cobweb model, delay differential equations and price stability.

1 Introduction

Maize is the number one crop in terms of area planted and accounts for 50-60% of total cereal production which then makes maize the second largest commodity crop in Ghana after cocoa and one of the most important crops for the country's agricultural sector and for food security [1]. Maize is the largest staple crop in Ghana and contributes significantly to consumer diets and according to a nationwide survey carried out in 1990 [2], 94% of all households had consumed maize during the period of the research. [3] also found that maize and maize-based foods account for 10.8% of household food expenditures by the poor and 10.3% of food expenditures by all other income groups. The per capita consumption of maize in Ghana was estimated to be 42.5 kg in the year 2000 and 943,000 Mt as national consumption in 2006 [4,5].

Maize is a food for an estimated 50 % of the population in sub-Saharan Africa and provides 50 % of their basic calories. This staple food has great nutritional values as it contains carbohydrate, protein, iron, oil, fibre, sugar, ash, vitamin B, and minerals. People in the sub-region consume maize as a starchy base in a wide variety of porridges, pastes, grits, and beer. Green maize (fresh on the cob) is eaten parched, baked, roasted or boiled and plays an important role in filling the hunger gap after the dry season [6]. As a result of this level of consumption, price instability is causing concerns in that increases in staple food prices have dire implications for the food security situation of the poor in Ghana.

Research has revealed that foodstuff prices exhibit high volatility with maize showing continual increasing price in recent years by as much as twenty three percent [7] and this assertion is buttressed by the findings of [8] that food prices for rice, maize and other cereals increased by 20 to 30 percent between the last few months of 2007 and the beginning of 2008.

According to [9], high food prices have radically different effects across countries and population groups such that the net food exporting countries benefit from improved terms of trade while net food importing countries, however, struggle to meet domestic food demand. The fact that, in Africa, most countries are net importers of cereals means therefore that these countries will be hard hit by rising prices. Higher food prices lead the poor to limiting their food consumption and shifting to less-balanced diets, with the consequential harmful effects on their health in the short and long runs. [10] also affirm that, higher food prices result in people eating less frequently and in lesser quantities, as well as cheaper and less nutritious food.

There has been a steady upward trend in recent times in food prices at a modest rate and a major contributor to this effect has been the increase in crude oil price, since a large increase in crude oil prices stands out among numerous factors to explain most of the jump in food prices over the last decade. Indeed, in a recent World Bank study, oil prices are found to be major contributing factor to food price increase than several other long-term price drivers, including exchange rates, interest rates and income [11]. Oil price increases exert both direct and indirect upward pressures on aggregate prices of food [9]. Prices of agricultural commodities in addition are affected by higher energy and chemical input prices, more frequent than usual adverse weather conditions, and the diversion of some food commodities to the production of bio-fuels; notably maize in the United States and edible oils in Europe. These conditions led global stock to use ratios of some grains down to levels not seen since the early 1970s [11].

Maize production is essentially performed by smallholder farmers under traditional tillage and rain-fed conditions which are increasingly erratic. Thus total market conditions for maize tend to follow the direct impact of these rainfall conditions on production. The situation frequently exacerbated by the poor or non-existent post-harvest management infrastructure such that even in periods of good moisture conditions, inefficient storage systems often result in price pressures arising from glut at harvest time and non-availability towards the end of the season [12] and according to [13], prices of maize, rice and yam, among others, shot up astronomically, few weeks after the Christmas festivities negatively affecting consumers.

The study by [14] found that tonnes of maize are rotting away in silos as a result of poor market; a situation so frustrating to farmers. For instance a maxi bag of maize was selling for GH¢32 compared to GH¢55 the previous year. Changes of food price are becoming increasingly relevant to producers and consumers in the competitive food markets in that price serves as an efficient means for seeking out production possibilities and potential, as well as allocating scarce resources within an economy [15,7]. It found that production delays in agricultural commodities are responsible for generating fluctuations in economic indicators. Therefore, a plausible mathematical setting in which to study these phenomena is provided by a delay-differential equation (DDE); in particular while the dependence of demand on price may be taken to be instantaneous, the supply term contains a delay [16].

This paper seeks to study the behaviour of the price dynamics of maize in the context of continuous-time cobweb models derived from linear and nonlinear delay differential equations. The models would be formulated based on real economic data of maize price demand and production in the Ashanti Region, of Ghana (the same data was used by [17]). It is intended to assist stakeholders to have knowledge of the price dynamics of maize so as to devise mechanisms to ensure food availability and the stabilization of prices. Having knowledge of the dynamics of food prices gives farmers the opportunity to make informed decisions concerning planting of a particular food crop, since planting decisions are taken based on the expected price during harvest.

1.1 Effects of Continuous-time Models

Cobweb models describe price dynamics in a market of a non-storable good that takes one time unit to produce. Thus cobweb model describes cyclical supply and demand in a market where the amount produced must be chosen before prices are observed [18].

Cobweb model of market stability is a discrete-time model that leads to a recurrence equation which describes the sequence of prices when the initial price is not the equilibrium value. One of the implicit assumptions in the demand and supply analysis is that suppliers decide the quantity of goods to be sent to market after they know the price of that good. In reality, however, most suppliers commit themselves to the supply decision before they know the price of the good in question. Thus, in reality, time is continuous although it is assumed that it evolves discretely under which framework the market is cleared once per fixed period of time [19]. It is therefore possible and reasonable to use continuous-time models and these models lead to differential equations rather than recurrence equations with price being given as a function of the continuously varying time parameter [20].

[21] applied a continuous time dynamic model of the short-term interest rate in finance and noted that the volatility of short-term rates is highly sensitive. [22] also found that the continuous-time model is preferable to the discrete and mixed-time models for the reason that it gives reasonable estimates with relatively few intervals while still making full use of the available information. Continuous-time linear models for some time series data, for instance, have been found valuable in certain applications, particularly in fitting models to irregularly spaced data [23,24]. Several recent papers [25,26,27,28] have investigated the use of continuous-time models, which are especially convenient for modelling irregularly spaced data when linear models are not adequate.

1.2 Effects of Nonlinear Models

Although a great deal of attention has been given to linear difference and differential equations, far less attention has been given to nonlinear relationships. However, this is now changing as many recent researches conducted in economics are considering nonlinearities. For example, [29] applied linear supply and nonlinear demand functions. [30,31] used linear demand functions with nonlinear supply equations. [32] also established a nonlinear model based on cobweb theory, where both the demand function and supply function were quadratic. These findings indicate that the nonlinear cobweb model may explain various irregular fluctuations observed in real economic data. [17] studied price dynamics of maize in Ghana using linear and nonlinear cobweb models which are constructed from real economic price and production data. It was deduced from their study that researchers rather use nonlinear models to avoid under or overestimation and make better predictions in real economic situations.

In economic systems with backward bending supply curves or multiple-valued demand curves, the dynamics described by the nonlinear at equilibrium points exhibit broad spectrums of complex behaviour. However, if the supply and demand curves are monotonic and single-valued, the behaviour remains qualitatively the same as in the linear case as only the three classes of behaviour are displayed: convergence to a fixed point, period-2 cycles, and instability [31].

The use of nonlinear dynamic models in economics and finance has expanded rapidly in the last two decades [33] and in studying the differences between linear and nonlinear models in engineering, [34] discovered that neglecting nonlinear effects can lead to serious errors as a nonlinear model can help one to avoid overdesign and build better products. [35] states, in other words, that whereas it takes longer time to run a nonlinear solution, yet it better replicates the actual physical system being analyzed and the test results will correlate very accurately.

1.3 Special Effects of DDEs

In considering the dynamics of price, production and, consumption of commodities, [36,37] proposed the price fluctuation models and under relatively mild conditions, determined the stability of equilibrium price. [38,39] considered naive consumer models and studied the oscillation of equilibrium price. However, these models were all based on ordinary differential equations. The lack of time lags (delay) in these models makes them quite different from the realistic problem.

In fact, a great deal of recent economic modelling has almost entirely ignored the potential role of delays in generating economic fluctuations and a good mathematical setting in which to consider this gap is provided by delay-differential equations [40,16]. While it is assumed that consumers base their buying decisions on the current market price, for most commodities there is actually a finite time τ that elapses before a change in production occurs. This time lag can be affected by several factors, such as the time necessary for increasing production, buying new machines, hiring workers or building factories; in the case of agricultural commodities, there are also natural constraints that affect the delay [37].

Delay Differential Equations (DDEs) are a special type of Functional Differential Equations (FDEs) in which there is the constraint that its time evolution can only depend on specific past values of the state variable at discrete or continuous times. DDEs arise in many applied models when traditional simplifications are abandoned for more realistic assumptions [41].

Delay differential equations have been widely used for many years in control theory and have recently become popular in biological and economical models. [42] employed differential system with delays similar to [43] to describe the price dynamics of two markets that are coupled via diffusive coupling terms. They studied two different time delay cases, namely when both markets experience the same time delay, and when the time delay is different across markets to determine their equilibrium and stability theoretically. The study also used numerical illustrations to confirm the theoretical findings. [44] used DDEs and realised that a broad spectrum of dynamic behaviours can be found in nonlinear delay differential equations. [45] developed a mathematical model for price cooperation with lag in which when the economic parameters satisfy some conditions, the existence and stability of periodic price are investigated.

2 Methods

In order to study price dynamics of maize in Ghana, records of price and production of maize from seventeen major market centres in the Ashanti Region were selected. This secondary data of maize are gathered from the year 1994 to 2013, from the Ministry of Food and Agriculture, Statistical Directorate Kumasi-Ghana as shown in Table 1. Linear and nonlinear delay differential equations of demand and supply are formulated from the data and then employed (equations) to derive cobweb models which are used for the study of price dynamics.

The Table 1 contains the quarterly price and production of maize data at the right and left sections of the table respectively. They are average price and production points across the seventeen (17) market centres. Find beneath details of the data contained in this table.

Table 2 shows the statistic values of the data in Table 1 which includes range, mean, skewness over 1 (moderate), kurtosis greater than 0 but within the expected value of 3 and their respective standard errors.

2.1 Continuous Time Linear Model

When the price of a good is *p*, the linear demand function of price is $D(p(t)) = a - \alpha p(t)$; this curve is generally negatively sloped-decreasing in mathematical term with a and α being positive constants. On the other hand the linear supply function of price with delay τ is positively sloped-increasing and is given by $S(p(t)) = b + \beta p(t-\tau)$, where b, β and τ are positive constants.

The models performed on the assumptions that, maize has no equal substitutes and there are no exogenous shocks needed to generate price fluctuations. The market price is determined by only the available supply in a single market and the rate of change of the price is proportional to the difference between supply and demand [46,47,37].

$$p'(t) = \left[D(p(t)) - S(p(t))\right] \quad (1)$$

This is a delay differential equation (DDE) that describes how the market price changes over time. It is assumed that when demand exceeds supply, price rises, and when supply exceeds demand it falls. It is also assumed that only the market price at time t- τ has an effect on the current supply price so that:

$$p'(t) = \left[(a-b) - \alpha p(t) - \beta p(t-\tau)\right], \text{ where } \tau>0, \text{ on } [0, b], b>0 \quad (2)$$

This equation with a single delay like all DDEs are mostly solved in a stepwise fashion with a principle called the method of steps: equation (2) would have initial function (also known as history function) as p(t) = s(t) defined over the interval [-τ, 0] and then its solution is mapped onto solutions of other functions. Thus the solution of this equation is going to be a mapping from functions on the interval [t-τ, t] into functions on the interval [t, t+τ],
[t+τ, t+2τ], etc., from time points t=0, τ, 2 τ,...

In other words, the solutions of this dynamical system can be considered as sequence of functions $p_0(t)$, $p_1(t)$, $p_2(t)$,.... defined over contiguous time interval of length τ [48]. However, in practice this boundary value problem is often solved numerically and in this study, MatLab solver dde23 would be used.

2.2 Continuous Time Non-linear Model

Considering a simple nonlinear delay differential equation (of quadratic form) for the supply function of price as,: $S(p(t)) = b + \beta p(t-\tau) - \delta p^2(t-\tau)$ and linear demand function of price as $D(p(t)) = a - \alpha p(t)$, where *a*, *b*, α, β and τ are positive constants. The supply is assumed to increase with increase in price until supply exceeds demand where the trend changes. We therefore have;

$$p'(t) = \left[(a-b) - \alpha p(t) - \beta p(t-\tau) + \delta p^2(t-\tau)\right] \text{ where, } \tau>0, \text{ on } [0, b], b>0 \quad (3)$$

Table 1. Price and production of maize in Ashanti region

Year	Prices of maize (Bag of 100 kg) in GH¢				Production of maize (Metric tons)				Total
	1st quarter	2nd quarter	3rd quarter	4th quarter	1st quarter	2nd quarter	3rd quarter	4th quarter	
1994	1.23	1.78	1.49	1.67	10198.13	14830.01	12403.34	13878.19	51309.67
1995	2.69	3.53	1.64	2.22	15947.12	20917.51	9730.17	13171.87	59766.67
1996	2.80	3.41	4.61	4.45	10152.87	12357.34	16710.75	16125.71	55346.67
1997	6.51	8.73	6.74	-	17268.13	23180.01	17886.86	-	58335.00
1998	5.06	5.62	4.43	4.26	19354.45	21514.8	16930.53	16287.52	74087.30
1999	4.64	4.94	5.76	4.60	15016.85	15978.78	18646.42	14875.28	64517.33
2000	9.32	11.05	9.90	10.39	14663.71	17381.97	15579.11	16342.87	63967.66
2001	13.87	18.76	12.98	13.04	13403.83	18127.28	12539.71	12595.85	56666.67
2002	14.83	15.38	10.97	11.36	25347.56	26300.98	18760.61	19417.51	89826.66
2003	14.03	17.06	16.32	14.49	14649.34	17818.76	17039.9	15132	64640.00
2004	17.80	21.67	23.46	23.06	12629.82	15372.77	16646.07	16362.01	61010.67
2005	29.67	45.02	34.25	28.56	11638.64	17660.19	13436.17	11203.67	53938.67
2006	25.91	27.39	20.98	18.87	15226.02	16097.1	12328.32	11090.56	54742.00
2007	25.85	32.34	26.41	26.06	13190.26	16500.28	13474.43	13296.04	56461.01
2008	32.34	56.66	56.72	49.39	10103.38	17700.44	17717.1	15428.41	60949.33
2009	60.79	71.77	55.13	52.21	15780.74	18631.95	14311.43	13552.55	62276.67
2010	53.29	55.51	52.76	46.82	21597.84	22497.63	21385.72	18976.81	84458.00
2011	55.10	75.41	81.34	89.95	10572.89	14470.08	15607.96	17260.74	57911.67
2012	110.69	124.44	90.97	76.00	18169.81	20426.27	14932.32	12475.61	66004.01
2013	77.09	76.24	74.86	81.67	17036.49	16847.17	16542.22	18047.12	68473.00

Table 2. Descriptive statistics

	N	Range	Mean		Std. Deviation	Skewness		Kurtosis	
	Statistic	Statistic	Statistic	Std. Error	Statistic	Statistic	Std. Error	Statistic	Std. Error
Price	79	123.21	30.2659	3.31709	29.48293	1.160	.271	.584	.535
Production	79	16570.81	16008.7172	383.99735	3413.04306	.556	.271	.604	.535
Valid N (listwise)	79								

This is simple nonlinear delay differential equation [31] and cannot have an exact solution in practice using an analytical method. Therefore, a numerical method is applied using MatLab solver dde23 with history function of $p(t) = s(t)$, on $[-\tau, 0]$.

3 Results and Discussion

This paper seeks to study the price dynamics of maize in the context of mathematical modelling using real economic data of maize price and production in Ashanti Region, Ghana. Modelling of various functions and their parameter estimates would be done by the use of SPSS and then the numerical solution of the delay differential equation run using MatLab.

3.1 Preliminary Analysis of Price and Production Data

The data are checked to correct any errors and then SPSS and MatLab used to verify the stationary status for both price and production data sets before formulating the demand and supply functions of price using regression analysis.

The below Fig. 1 shows the time series plot (behaviour) of price. It indicates that price data is non-stationary and also exhibits nonlinearity characteristics in the form close to quadratic.

3.2 Parameter Estimates

The following Table 3 contains the coefficients of the parameters estimated from the data analyzed. All the values are statistically significant.

Table 3. Coefficients of demand and supply functions

Model	**Unstandardized coefficients**		**Standardized coefficients**	t	**sig.**	
	B	**Std. error**	**Beta**			
1. Price	-96.16	20.26	-0.49	-4.75	0.00	Demand function
a. Dependent Variable: Production			b. Linear Regression through the Origin			
2. Price	354.28	36.89	1.72	9.60	0.00	Supply function
Price**2	-2.78	0.45	-1.10	-6.14	0.00	
3. Price	167.99	43.23	0.42	3.89	0.00	
a. Dependent Variable: Production_Lag			b. Linear Regression through the Origin			

The Table 3 above contains the parameter values of demand and supply functions. The regression was carried out through the origin because in both cases the intercepts were not statistically significant. It also contains parameter values of linear demand function of price (1), while (2) is parameter values for quadratic supply function of price and (3), linear supply function of price.

3.3 Demand Function of Price

The demand function of price from the Table 3 is given below with its parameter estimates checked to be statistically significant. This function was obtained from price data of order two (2) differencing and production data of order one (1) differencing.

$$D(p(t)) = -96.162\, p(t) \text{ where a = 0} \quad (4)$$

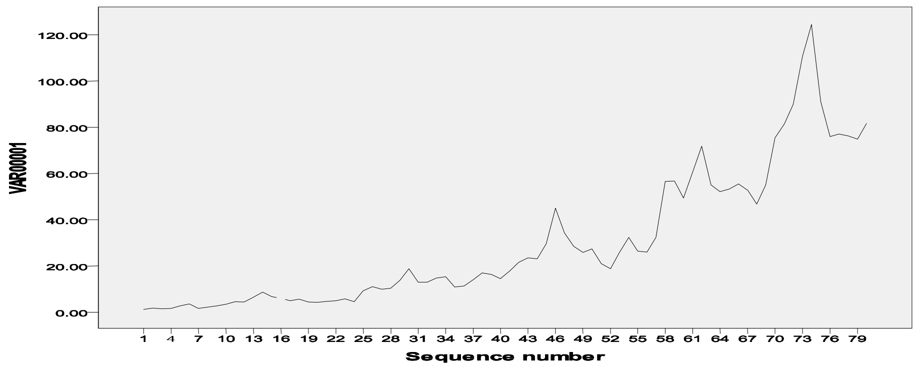

Fig. 1. Time series plot of price

3.4 Supply Function of Price

Similarly supply functions of price with time delay τ are given below. Equation (5) was obtained from price data of order one (1) differencing and production data of order two (2) differencing. However, equation (6) was obtained with no order of differencing.

$$S(p(t)) = 167.994\, p(t-\tau) \text{ where b = 0} \quad (5)$$

$$S(p(t)) = 354.28\, p(t-\tau) - 2.782\, p^2 (t-\tau) \text{ where b = 0} \quad (6)$$

The delay τ expresses time that is needed to realise change of supply in dependence on trend of price. Thus current production depends on the past price.

3.4.1 Analysis of linear model

From equations (4) and (5), the rate of change of price is given by the following equation, which is in the form of equation (2) where $\tau=1$:

$$\frac{1}{96.16} P'(t) = -P(t) - 1.75P(t-1) \quad (7)$$

This is a delay differential equation which can be solved using MatLab solver dde23 (DDE code attached at the appendix). Equation (7) is divided by 96.162 so as to make the solution very smooth when it is run. The history function is set at *P(t)*=1.23 (initial price from Table 1), when $t \le 0$, with (7) on the interval [0, 100]. By using a numerical approach, the solution of equation (7) is presented in graphical form (Fig. 2) below.

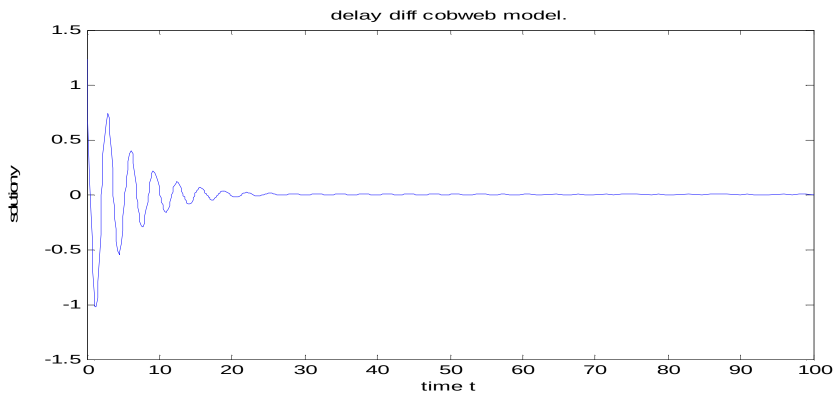

Fig. 2. Oscillation of price around equilibrium

It is clear from the Fig. 2, above that the solution of equation (7) oscillates and tends to an equilibrium price of zero with time. However, this equilibrium price is unrealistic, due to the fact that producers are sensitive towards price and, moreover, there is insufficiency of food supply.

3.4.2 Analysis of nonlinear model

From equations (4) and (6), the rate of change of price is given by the following equation, which is in the form of equation (3) where τ=1:

$$\tfrac{1}{96.162}P'(t) = -P(t) - 3.68P(t-1) + 0.029P^2(t-1) \tag{8}$$

Based on same assumption for smoothness of the solution using MatLab solver dde23, equation (8) divided by 96.162 and the history function set at *P(t)*=1.23, when t≤0, with (8) also on the interval [0, 100] just as in the linear case. The solution of equation (8) in numerical form is presented graphically as follows (Fig. 3 below):

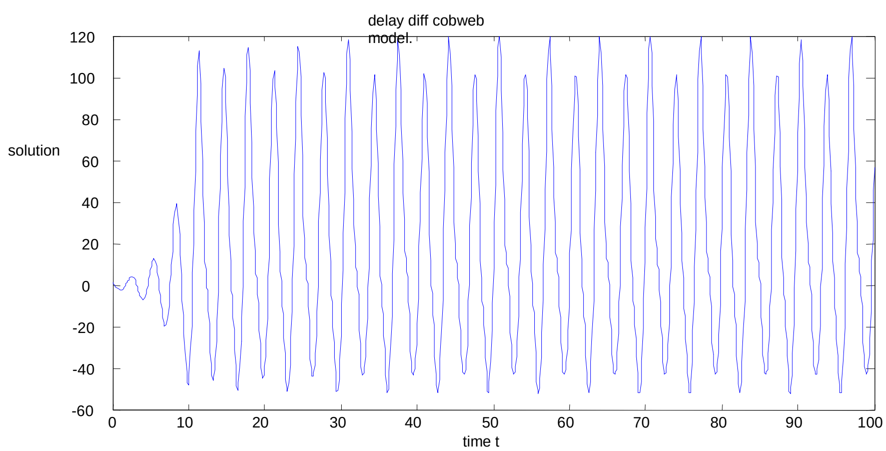

Fig. 3. Oscillations of price around equilibrium

It is clear from the Fig. 3, above that the solution of equation (8) oscillates between two (2) equilibrium (price) points and would never converge to either equilibrium price due perhaps to inflation, food insufficiency and/or producers' sensitivity towards price.

3.5 Asymmetric Effects

The effects of delay parameter τ on oscillations are discussed. The phenomenon of time delay τ can suppress or increase fluctuations of price [49,40,16].

The oscillations (fluctuations) are suppressed for $\tau \leq 0.5$**,** as shown in Fig. 4, using equation (8). This indicates that price would be in stable equilibrium when factors affected by time lag in the system are improved.

The price oscillations (fluctuations) start to increase as seen in fig. 5, and with time become asymmetric (just like fig. 3) about the equilibrium for $\tau > 0.5$ using equation (8). This indicates that price would never be in stable equilibrium until factors affected by time lag, such as the time necessary for increasing supply, buying new inputs, hiring workers or transporting commodities to market centres, building warehouse and reducing effects of natural constraints are improved [37]. On the contrary, equation (7), for $\tau > 0.5$, would still be in stable equilibrium.

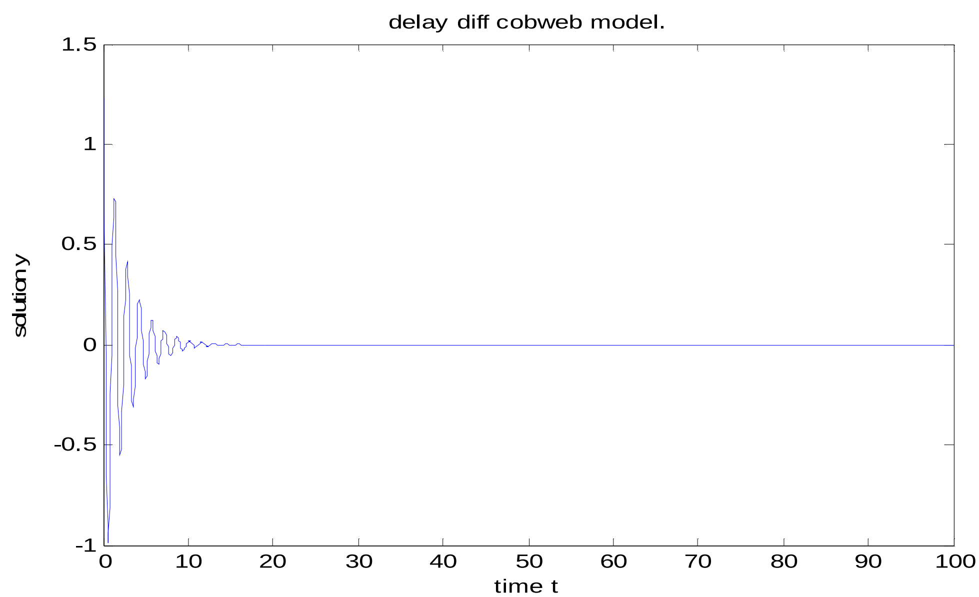

Fig. 4. Oscillation of price equilibrium with delay $\tau \leq 0.5$

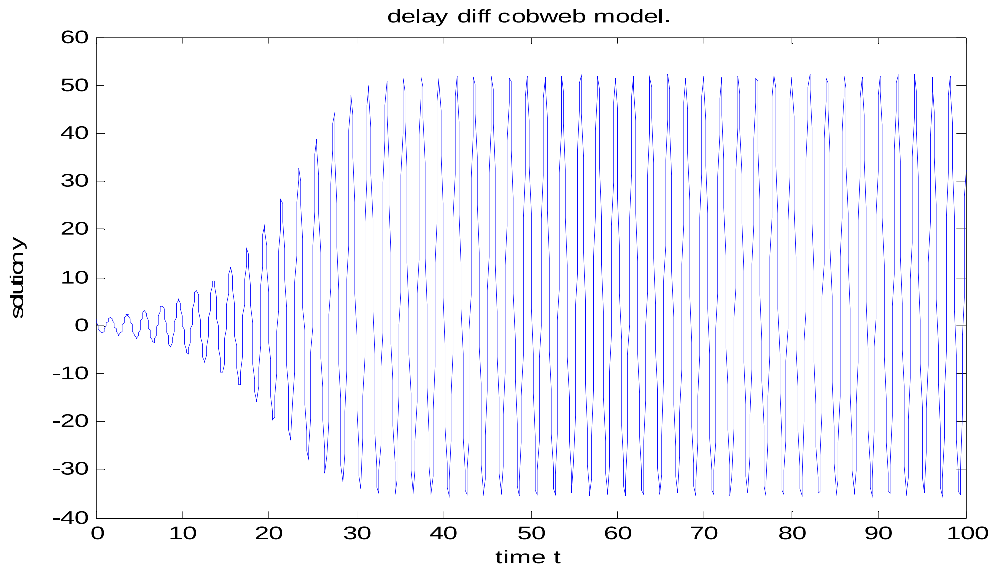

Fig. 5. Oscillation of price equilibrium with delay τ >0.5

4 Conclusion

This paper seeks to study the price dynamics of maize in Ghana, in the context of mathematical modelling using a continuous-time cobweb model derived from linear and nonlinear delay differential functions of price. The stability conditions of two cobweb models: continuous-time linear and nonlinear models are discussed.

The models performed on the assumptions that, maize has no equal substitutes and there are no exogenous shocks needed to generate price fluctuations so that market price would be determined by only the available supply in a single market.

From the results of the analysis performed on the real economic price and production data, using MatLab dde23 solver, the nonlinear model (formulated from linear demand and quadratic supply functions) showed oscillations between and around two equilibrium price points and would neither converge with time. The oscillations observed from this solution are realistic and a reflection of real market price situation in Ghana. Therefore no equilibrium price would be achieved due to producers' sensitivity towards price. Inflation and/or food insufficiency can also contribute to this price instability.

The linear model (formulated from linear demand and linear supply functions) on the other hand showed little oscillations and converged to stable zero equilibrium price point which is unrealistic. Effects of delay parameter τ on oscillations are also discussed. It is observed that oscillations (price fluctuations) are suppressed for $\tau \leq 0.5$**,** using the nonlinear delay equation. This is an indication that price fluctuations are reduced, if and only if, factors affected by time lag, such as the time necessary for increasing supply, buying new inputs, hiring workers or transporting commodities to market centres, building warehouse and reducing effects of natural constraints on crop yields are improved.

On the contrary, the linear delay differential equation, for $\tau > 0.5$ or $\tau \leq 0.5$, would still be in stable equilibrium.

Acknowledgements

The authors are grateful to Mr Akiomi and Ben (Ministry of Food and Agriculture Statistical Directorate, Ashanti Region Kumasi - Ghana) for making available records which enabled, us to extract the price and production figures of maize.

Competing Interests

Authors have declared that no competing interests exist.

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APPENDIX

DDE code

```
Function ddeprice1
%   DDEPRICE  excercise for DDE23.
%   This is cobweb model with delay differential equations (DDEs)
%   technigue.
%   ddeprice are solved on [0, 100] with history y_1(t) = 1.23,for t <= 0.
%   The lags are specified as a vector [1], the delay differential
%   equations are coded in the subfunction DDEPRICE, and the history is
%   evaluated by the function histprice. Because the history is constant it
%   could be supplied as a vector:
%   sol = dde23(@ddeprice,[1],1.23,[0, 100]);
sol = dde23(@ddeprice,1,@histprice,[0, 100]);
figure;
plot(sol.x,sol.y)
title('delay diff cobweb model.');
xlabel('time t');
ylabel('solution y');
% -----------------------------------------------------------------------
function s = histprice(t)
% Constant history function for ddeprice.
s = (1.23);
% -----------------------------------------------------------------------
function dydt = ddeprice(t,y,Z)
% Delay Differential equations function for DDEPRICE.
ylag1 = Z(:,1);
dydt = -1*y(1)-1.75*ylag1(1);
```

*The same code used for the nonlinear with few changes in code

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