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Improved and Robust Estimators for Finite Population Variance Using Linear Combination of Probability Weighted Moment and Quartiles as Auxiliary Information

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Authors' contributions

This work was carried out in collaboration between all authors. Author MAB designed the study, performed the statistical analysis, wrote the protocol and wrote the first draft of the manuscript. Authors SM, YI and SAS managed the analyses of the study. Author MS managed the literature searches. All authors read and approved the final manuscript.

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ABSTRACT

By adapting the scheme of modification technique, we have developed some new ratio type variance estimators to estimate the finite population variance for skewed populations under survey. We have also derived the functions of efficiency to intensify the quality of efficacy of newly developed estimators. The bias and mean square error of present and newly developed estimators has been worked out through numerical problem to determine the improvement of newly developed estimators over present estimators.

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1. INTRODUCTION

The scheme of modifying the estimators to develop new estimators in theory of variance estimation has achieved its scope in every scientific field. In most of the surveys, sometimes two variants such as auxiliary variable *(X)* and Study variable *(Y)* about whom we sought out the information makes the strong association with one another. Where researchers have always taken this advantage of contributing their ideas to develop estimators for the increment of precision, efficiency and accuracy. Various authors who have contributed their creative ideas to produce efficient estimators for survey estimation. Among them some are most famous in the field of survey sampling like Murthy [1], Isaki [2], Upadhyaya and Singh [3], Kadilar & Cingi [4] and Subramani, J and Kumarapandiyan, G [5] These authors have used auxiliary information in the modification of estimators to increase the efficiency of newly modified estimators. Recently improvements to ratio estimators in theory of survey estimation contributed by authors such as Jeelani. Iqbal and Maqbool. S [6], Bhat et al. [7] have also utilized this auxiliary information in different forms to enhance the improvement of proposed estimators.

Let the finite population under survey is $G = G_1, G_2 ......G_N$, which consists of *N* distinct and identifiable units. Let *Y* is variable of interest about which information is to be sought out on the bases of samples Y_i, i=1,2,3…drawn from finite population $G_i, i = 1,2,3........N$. To estimate the finite variance population parameter denoted by $S_Y^2 = \frac{1}{N-1}\sum_{I=1}^{N}\left(y_i - \bar{y}\right)^2$ from $G = G_1, G_2 ......G_N$, we use the technique of without simple Random Sampling Scheme for the estimation of population variance. In this paper, our aim is to achieve the precise and reliable estimates when the population under investigation is non-normal or skewed because using non-conventional measures as auxiliary information avoids the concern of outliers in the survey data.

2. MATERIALS AND METHODS

2.1 Notations Used for Descriptive Statistics of Auxiliary Variable *(X)* and Study Variable *(Y)*

$N =$ Population size. $n =$ Sample size. $\gamma = \frac{1}{n}$,*Y*= study variable. *X*= Auxiliary variable $\bar{X}, \bar{Y}$ = Population means. $\bar{x}$, $\bar{y} =$ Sample means. S_Y^2 , $S_x^2 =$ Population variances s_y^2 , s_x^2 = sample variances. C_x , $C_y =$ Coefficients of variation. $\rho =$ Correlation coefficient. $\beta_{1(x)} =$ Skewness of the auxiliary variable, $\beta_{2(x)} =$ Kurtosis of the auxiliary variable. $\beta_{2(y)} =$ Kurtosis of the study variable. B(.)=Bias of the estimator. MSE(.)= Mean square error. $\hat{S}_R^2 =$ Ratio type variance estimator, $Q_a =$ quartile average $Q_d =$ quartile deviation, Q_1= first quartile, Q_2= second quartile, Q_3= third quartile, Q_r= quartile range.

2.2 Existing Estimators from the Literature

2.2.1 Variance estimator from the literature given by Isaki

$$\hat{S}_{Is}^2 = s_y^2\left[\frac{S_x^2}{s_x^2}\right] \tag{1}$$

The bias and mean square error of the estimator is given as

$$\text{Bias } (\hat{S}_{IS}^2) = \gamma S_y^2 \left[\left(\beta_{2(x)} - 1\right) - (\lambda_{22} - 1)\right] \tag{2}$$

$$\text{MSE} (\hat{S}_{IS}^2) = \gamma S_y^4 \left[\left(\beta_{2(y)} - 1\right) + (\beta_{2(x)} - 1) - 2(\lambda_{22} - 1)\right] \tag{3}$$

2.2.2 Variance estimator from the literature given by Kadilar and Cingi

$$\hat{S}^2_{kc1} = s_y^2\left[\frac{S_x^2 + C_x}{s_x^2 + C_x}\right] \tag{4}$$

The bias and mean square error of the estimator is given as

$$\text{Bias}\,(\hat{S}^2_{kc1}) = \gamma S_y^2 A_1 \left[A_1\left(\beta_{2(x)} - 1\right) - (\lambda_{22} - 1)\right] \tag{5}$$

$$\text{MSE}\,(\hat{S}^2_{kc1}) = \gamma S_y^4 \left[\left(\beta_{2(y)} - 1\right) + A_1^2\,(\beta_{2(x)} - 1) - 2A_1(\lambda_{22} - 1)\right] \tag{6}$$

2.2.3 Variance estimator from the literature given by Subramani and Kumarapandiyan

$$\hat{S}^2_{jG} = s_y^2\left[\frac{S_x^2 + Q_1}{s_x^2 + Q_1}\right] \tag{7}$$

The bias and mean square error of the estimator is given as

$$\text{Bias}\,(\hat{S}^2_{jG}) = \gamma S_y^2 A_{jG} \left[A_{jG}\left(\beta_{2(x)} - 1\right) - (\lambda_{22} - 1)\right] \tag{8}$$

$$\text{MSE}\,(\hat{S}^2_{jG}) = \gamma S_y^4 \left[\left(\beta_{2(y)} - 1\right) + A_{jG}^2\,(\beta_{2(x)} - 1) - 2A_{jG}(\lambda_{22} - 1)\right] \tag{9}$$

2.2.4 New modified and developed estimators

$$\hat{S}^2_{MS1} = s_y^2\left[\frac{S_x^2 + (S_{pw} + Q_1)}{s_x^2 + (S_{pw} + Q_1)}\right] \quad \hat{S}^2_{MS2} = s_y^2\left[\frac{S_x^2 + (S_{pw} + Q_2)}{s_x^2 + (S_{pw} + Q_2)}\right] \quad \hat{S}^2_{MS3} = s_y^2\left[\frac{S_x^2 + (S_{pw} + Q_3)}{s_x^2 + (S_{pw} + Q_3)}\right]$$

$$\hat{S}^2_{MS4} = s_y^2\left[\frac{S_x^2 + (S_{pw} + Q_r)}{s_x^2 + (S_{pw} + Q_r)}\right] \quad \hat{S}^2_{MS5} = s_y^2\left[\frac{S_x^2 + (S_{pw} + Q_d)}{s_x^2 + (S_{pw} + Q_d)}\right] \quad \hat{S}^2_{MS6} = s_y^2\left[\frac{S_x^2 + (S_{pw} + Q_a)}{s_x^2 + (S_{pw} + Q_a)}\right]$$

Derivation technique to work out the bias and mean square error of the new estimators $\hat{S}^2_{MSi};\ i = 1,2,....,6$ Where' M' denotes Mushtaq and 'S' denotes Showkat

$e_0 = \dfrac{s_y^2 - S_y^2}{S_y^2}$ $e_1 = \dfrac{s_x^2 - S_x^2}{S_x^2}$ Further, we can write $s_y^2 = S_y^2(1 + e_0)$ $s_x^2 = S_x^2(1 + e_0)$ and from the definition of e_0 and e_1 we obtain:

$$E[e_0] = E[e_1] = 0 \quad E[e_0^2] = \frac{1-f}{n}(\beta_{2(y)} - 1) \quad E[e_1^2] = \frac{1-f}{n}(\beta_{2(x)} - 1), \quad E[e_0 e_1] = \frac{1-f}{n}(\lambda_{22} - 1)$$

The proposed estimator $\hat{S}^2_{MSi};\ i = 1,2,3,r,d,a$ is given below:

$$\hat{S}^2_{MSi} = s_y^2\left[\frac{S_x^2 + \alpha a_i}{s_x^2 + \alpha a_i}\right]$$

$$\Rightarrow \quad \hat{S}^2_{MSi} = s_y^2(1 + e_0)\left[\frac{S_x^2 + \alpha a_i}{s_x^2 + e_1 S_x^2 + \alpha a_i}\right] \quad \Rightarrow \quad \hat{S}^2_{MSi} = \frac{S_y^2(1 + e_0)}{(1 + A_{MSi} e_1)}$$

Where $A_{MSi} = \dfrac{S_x^2}{S_x^2 + \alpha a_i}$, $a_i = (S_{pw} + Q_i);\ i = 1,2,3,r,d,a$

$$\Rightarrow \quad \hat{S}^2_{MSi} = S_y^2(1 + e_0)(1 + A_{MSi} e_1)^{-1} \quad \Rightarrow \quad \hat{S}^2_{MSi} = S_y^2(1 + e_0)(1 - A_{MSi} e_1 + A_{MSi}^2 e_1^2 - A_{MSi}^3 e_1^3 + ...)$$

Expanding and neglecting the terms more than 3rd order, we get

$$\hat{S}^2_{MSi} = S^2_y + S^2_y e_0 - S^2_y A_{MSi} e_1 - S^2_y A_{MSi} e_0 e_1 + S^2_y A^2_{MSi} e^2_1$$

$$\Rightarrow \hat{S}^2_{MSi} - S^2_y = S^2_y e_0 - S^2_y A_{MSi} e_1 - S^2_y A_{MSi} e_0 e_1 + S^2_y A^2_{MSi} e^2_1 \quad (10)$$

By taking expectation on both sides of (5), we get

$$E(\hat{S}^2_{MSi} - S^2_y) = S^2_y E(e_0) - S^2_y A_{MSi} E(e_1) - S^2_y A_{MSi} E(e_0 e_1) + S^2_y A^2_{MSi} E(e^2_1)$$
$$Bias(\hat{S}^2_{MSi}) = S^2_y A^2_{MSi} E(e^2_1) - S^2_y A_{MSi} E(e_0 e_1)$$
$$Bias(\hat{S}^2_{MSi}) = \gamma S^2_y A_{MSi} [A_{MSi} (\beta_{2(x)} - 1) - (\lambda_{22} - 1)] \quad (11)$$

Squaring both sides of (5) and (6), neglecting the terms more than 2nd order and taking expectation, we get

$$E(\hat{S}^2_{MSi} - S^2_y)^2 = S^4_y E(e^2_0) + S^4_y A^2_{MSi} E(e^2_1) - 2 S^4_y A_{MSi} E(e_0 e_1)$$
$$MSE(\hat{S}^2_{MSi}) = \gamma S^4_y [(\beta_{2(y)} - 1) + A^2_{MSi} (\beta_{2(x)} - 1) - 2 A_{MSi} (\lambda_{22} - 1)] \quad (12)$$

2.3 Efficiency Conditions of Proposed Estimators with Existing Estimators

We have derived the efficiency conditions of proposed estimators with the existing estimators under which proposed estimators $\hat{S}^2_P (P = 1,2,3........)$ are performing better than the existing estimators.

The bias and Mean square error of existing ratio type estimators up to the first order of approximation is given by

$$Bias(\hat{S}^2_K) = \gamma S^2_y R_K [R_K (\beta_{2x} - 1) - (\lambda_{22} - 1)] \quad (13)$$

$$MSE(\hat{S}^2_K) = \gamma S^4_y \begin{bmatrix} (\beta_{2y} - 1) + R^2_K (\beta_{2x} - 1) \\ - 2 R_K (\lambda_{22} - 1) \end{bmatrix} \quad (14)$$

Where R_k =Existing constant, K=1,2.3…..

Bias, MSE and constant of proposed estimators is given by

$$Bias(\hat{S}^2_P) = \gamma S^2_y R_P [R_P (\beta_{2x} - 1) - (\lambda_{22} - 1)] \quad (15)$$

$$MSE(\hat{S}^2_P) = \gamma S^4_y [(\beta_{2y} - 1) + R^2_P (\beta_{2x} - 1) - 2 R_P (\lambda_{22} - 1)] \quad (16)$$

Where R_p =Proposed constant, P=1,2,3…….

From Equation (2) and (3), we have

$$MSE(\hat{S}^2_P) \leq MSE(\hat{S}^2_K) if \lambda_{22} \geq 1 + \frac{(R_P + R_K)(\beta_{2x} - 1)}{2}$$

$$MSE(\hat{S}^2_P) \leq MSE(\hat{S}^2_K)$$

$$\gamma S^4_y [(\beta_{2y} - 1) + R^2_P (\beta_{2x} - 1) - 2 R_P (\lambda_{22} - 1)] \leq \gamma S^4_y [(\beta_{2y} - 1) + R^2_K (\beta_{2x} - 1) - 2 R_K (\lambda_{22} - 1)] \quad (17)$$

$$\Rightarrow [(\beta_{2y} - 1) + R^2_P (\beta_{2x} - 1) - 2 R_P (\lambda_{22} - 1)] \leq [(\beta_{2y} - 1) + R^2_K (\beta_{2x} - 1) - 2 R_K (\lambda_{22} - 1)] \quad (18)$$

$$\Rightarrow \left[1+R_P^2\left(\beta_{2x}-1\right)-2R_P\left(\lambda_{22}-1\right)\right] \leq \left[1+R_K^2\left(\beta_{2x}-1\right)-2R_K\left(\lambda_{22}-1\right)\right] \tag{19}$$

$$\Rightarrow \left(\beta_{2x}-1\right)\left(R_P^2-R_K^2\right)\left[-2R_P\left(\lambda_{22}-1\right)\right] \leq \left[-2R_K\left(\lambda_{22}-1\right)\right] \tag{20}$$

$$\Rightarrow \left(\beta_{2x}-1\right)\left(R_P^2-R_K^2\right)\left[-2\left(\lambda_{22}-1\right)\left(R_P-R_K\right)\right] \leq 0 \tag{21}$$

$$\Rightarrow \left(\beta_{2x}-1\right)\left(R_P^2-R_K^2\right) \quad \leq \left[2\left(\lambda_{22}-1\right)\left(R_P-R_K\right)\right] \tag{22}$$

$$\Rightarrow \left(\beta_{2x}-1\right) \leq \frac{2\left(\lambda_{22}-1\right)\left(R_P-R_K\right)}{\left(R_P^2-R_K^2\right)} \tag{23}$$

$$\Rightarrow \left(\beta_{2x}-1\right) \leq \frac{2\left(\lambda_{22}-1\right)\left(R_P-R_K\right)}{\left(R_P-R_K\right)\left(R_P+R_K\right)} \tag{24}$$

$$\Rightarrow \left(\beta_{2x}-1\right)\left(R_P+R_K\right) \leq 2\left(\lambda_{22}-1\right) \tag{25}$$

By solving equation (20), we get

$$MSE\left(\hat{S}_P^2\right) \leq MSE\left(\hat{S}_K^2\right) if \lambda_{22} \geq 1+\frac{\left(R_P+R_K\right)\left(\beta_{2x}-1\right)}{2}$$

3. NUMERICAL ILLUSTRATION

We use the data of Murthy (1967) [5] page 228 in which fixed capital is denoted by *X*(auxiliary variable) and output of 80 factories are denoted by *Y*(study variable).we apply the proposed and existing estimators to this data set and the data statistics are given below:

$$N = 80, n = 20, S_x = 8.4542, C_x = 0.7507,$$
$$\overline{X} = 11.2624, \beta_{2x} = 2.8664, \overline{Y} = 51.8264,$$
$$\beta_{2y} = 2.2667, \rho = 0.9413, \beta_{1x} = 1.,05,$$
$$\lambda_{22} = 2.2209, S_y = 18.3569, Q_1 = 7.5750,$$
$$Q_2 = 9.318, Q_3 = 16.975, Q_a = 11.0625$$
$$Q_d = 5.9125, Q_r 11.825, S_{pw} = 7.9136$$

Bias and mean square error of the existing and the proposed estimators

Estimators	Bias	Mean square error
Isaki [2]	10.8762	3925.1622
Kadilar & Cingi [4]	10.4399	3850.1552
Subramani & Kumarapandiyan [5]	6.1235	3180.7740
Proposed (MS1)	**2,8816**	**2175.5261**
Proposed (MS2)	**3.2507**	**2216.4052**
Proposed (MS3)	**1.5314**	**2054.1669**
Proposed (MS4)	**2.3922**	**2126.1308**
Proposed (MS5)	**3.6306**	**2261.1165**
Proposed (MS6)	**2.5352**	**2139.7572**

Percent relative efficiency of proposed estimators with existing estimators

Estimators	P1	P2	P3	P4	P5	P6
Isaki [2]	180.42	177.09	191.08	184.61	173.59	183.43
Kadilar & Cingi [4]	176.97	173.71	187.43	181.08	170.27	179.93
Subramani & Kumarapandiyan [5]	146.20	143.51	154.84	149.60	140.67	148.65

4. CONCLUSION

Use of non-conventional measures in this study to enhance the improvement of new estimators has clearly identified the improvement of new estimators which can be seen from the table of percent relative efficiency and also from the table of bias and MSE of existing and proposed estimators. Hence the proposed estimator may be preferred over existing estimators for use in practical applications.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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