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Intuitionistic Fuzzy Assignment Problem: An Application in Agriculture

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Authors' contributions

This work was carried out in collaboration between all authors. All authors read and approved the final manuscript.

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ABSTRACT

Assignment problem is a well known and most suited for solving real world problems. This paper deals with Intuitionistic fuzzy assignment problem whose cost $\tilde{C}_{ij}$ has been considered as an Intuitionistic triangular fuzzy numbers. By defuzzifying, the assignment costs are converted into crisp values and the optimum solution is obtained by using Branch and Bound method.

Keywords: Assignment problem; intuitionistic triangular fuzzy numbers; branch and bound method; optimal allocation.

1. INTRODUCTION

Assignment problem is a special kind of transportation problem in which each source should have the capacity to fulfill the demand of any of the destinations. Assignment problem plays an important role in solving problems of engineering, agriculture and management sciences. The objective of assignment problem is to assign a number of origins (jobs) to the equal

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number of destinations (persons) at a minimum cost or maximum profit. Various algorithms / methods like Hungarian method, neural networks, genetic algorithm have been developed to obtain the optimal solution of the assignment problems. The first mathematical formulation of fuzziness was pioneered by [1]. [2] made a numerous attempts to explore the ability of fuzzy set theory to become a useful tool for adequate mathematical analysis of real world problems. [3] introduced Intuitionistic fuzzy sets as an extension of Zadeh's notion of fuzzy set. Fuzzy methods have been developed in virtually all branches of decision making problems can be found in [4,5,6]. [7] solve the FLPP with fuzzy variables in parametric form. [8] proved some theorems and proposed a fuzzy assignment model that considers all individuals to have same skills. [9] introduce the concepts of fuzzy set theory into the decision-making problems involving uncertainty and imprecision. [10] investigated Fuzzy generalized assignment problem with credibility Constraints. Different kinds of fuzzy assignment problems and Intuitionistic Fuzzy assignment problems has been discussed by several authors (see, [11,12, 13,14]). Ranking and defuzzification methods based on area compensation fuzzy sets and systems can be found in [15]. Ranking of trapezoidal Intuitionistic fuzzy numbers was presented [16]. The Hungarian method for the assignment and transportation problems given by [17]. In this paper assignment costs has been considered as Intuitionistic fuzzy numbers which are more realistic and general in nature. The assignment costs are converted into crisp values by defuzzifying with the accuracy function and the optimum solution is obtained by using Branch and Bound method.

2. PRELIMINARIES

2.1 Fuzzy Set

Let A be a classical set, $\mu_{\overline{A}}(x)$ be a function from A to [0, 1]. A fuzzy set $\overline{A}$ with the membership function $\mu_{\overline{A}}(x)$ is defined by $\overline{A} = \{(x, \mu_{\overline{A}}(x)); x \in A, \mu_{\overline{A}}(x) \in [0,1]\}$.

2.1.1 Intuitionistic fuzzy set (IFS)

Let X denote universe of discourse, then an intuitionistic fuzzy set $\overline{A}^I$ in X is given by $\overline{A} = \{(x, \mu_{\overline{A}^I}(x), v_{\overline{A}^I}(x)); x \in X,]\}$ where, $\mu_{\overline{A}^I}(x), v_{\overline{A}^I}(x))$: X→[0,1] are functions such that $0 \le (\mu_{\overline{A}^I}(x) + v_{\overline{A}^I}(x)) \le 1$ for all x∈X. For each x the member ship function $\mu_{\overline{A}^I}(x)$ and $v_{\overline{A}^I}(x)$ represent the degree of membership and non-membership of the element x∈X to A⊂X respectively.

2.1.2 Intuitionistic fuzzy number (IFN)

An Intuitionistic fuzzy set of real line R is called an Intuitionistic fuzzy number if the following holds:

(i). There exists $x_o \in R$, $\mu_{\overline{A}^I}(x_o)$ = 1 and $v_{\overline{A}^I}(x_o)$ = 0, x_o is called the mean value of $\overline{A}^I$.

(ii) $\mu_{\overline{A}^I}$ is a continuous mapping from R to the closed interval [0,1] and for all x∈R, the relation $o \le \mu_{\overline{A}^I} + v_{\overline{A}^I} \le 1$ holds.

2.1.3 Triangular intuitionistic fuzzy number (TrIFN)

A triangular Intuitionistic fuzzy number $\overline{A}^I$ is an Intuitionistic fuzzy subset in R with the following membership function $\mu_{\overline{A}^I}(x)$ and non-membership function $v_{\overline{A}^I}(x)$.

$$\mu_{\overline{A}^I}(x) = \begin{cases} 0, & x < a_1 \\ \dfrac{(x-a_1)}{a_2-a_1}, & a_1 \le x \le a_2 \\ \dfrac{(a_3-x)}{a_3-a_2}, & a_2 \le x \le a_3 \\ 0 & x > a_3 \end{cases}$$

$$v_{\overline{A}^I}(x) = \begin{cases} 1, & x < a_1' \\ \dfrac{(a_2-x)}{a_2-a_1'}, & a_1' \le x \le a_2 \\ \dfrac{(x-a_2)}{a_3'-a_2}, & a_2 \le x \le a_3' \\ 1 & x > a_3' \end{cases}$$

Where $a_1' \le a_1 \le a_2 \le a_3 \le a_3'$ and ($\mu_{\overline{A}^I}(x)$, $v_{\overline{A}^I}(x)) \le 0.5$, $\mu_{\overline{A}^I}(x)$ = $v_{\overline{A}^I}(x)$ for all x∈R.

The ***TrIFN*** is given by
$\overline{A}^I = (a_1, a_2, a_3; a_1^1 a_2, a_3^1)$.

3. DEFUZZIFICATION

We define accuracy function to defuzzify a given triangular Intuitionistic fuzzy number is

$$H(\overline{a}^I) = \frac{(a_1 + 2a_2 + a_3) + (a_1^{'} + 2a_2 + a_3^{'})}{8}$$

3.1 Intuitionistic Fuzzy Assignment Problem (IFAP)

Suppose there are n jobs which is to be performed and n persons are available for doing these jobs. Assume that each person's can do each job at a time, though with different. Let $\tilde{C}_{ij}$ be an Intuitionistic fuzzy cost of assigning $i^{th}$the person to the j^{th}job. Let the decision variable y_{ij} denoting he assignment of the $i^{th}$the person to the j^{th}job. The problem is to find an assignment (which job should be assigned to which person on one – one basis) so that total cost of performing all jobs is minimum. Problems of this kind are known as assignment problem. Mathematically an IFAP is given below:

$$\text{Minimize } Z = \sum_{j=1}^{n} \sum_{j=1}^{n} \tilde{C}_{ij} X_{ij}$$

Subject to

$$\sum_{i=1}^{n} X_{ij} = 1 \qquad for\ i = 1, 2, \ldots, n.$$

$$\sum_{j=1}^{n} X_{ij} = 1 \qquad for\ j = 1, 2, \ldots, n.$$

Where

$$X_{ij} = \begin{cases} 1, if\ the\ \ i^{th}\text{the crop is assigned to the } j^{th}\text{paddock} \\ 0, \quad if\ the\ \ i^{th}\text{the crop is not assigned to the } j^{th}\text{paddock} \end{cases}$$

$$\tilde{C}_{ij}^{I} = (C_{ij}^1, C_{ij}^2, C_{ij}^3)(C_{ij}^{1'}, C_{ij}^2, C_{ij}^{3'})$$

Crops	Paddocks					
	1	2	⋯	j	⋯	n
1	$\tilde{a}_{11}$	$\tilde{a}_{12}$	…	$\tilde{a}_{1j}$	…	$\tilde{a}_{1n}$
2	$\tilde{a}_{21}$	$\tilde{a}_{21}$	…	$\tilde{a}_{2j}$	…	$\tilde{a}_{2n}$
⋮	⋮	⋮		⋮	⋮	
i	$\tilde{a}_{i1}$	$\tilde{a}_{i1}$	…	$\tilde{a}_{ij}$	…	$\tilde{a}_{jn}$
⋮	⋮	⋮		⋮	⋮	
n	$\tilde{a}_{n1}$	$\tilde{a}_{n2}$	…	$\tilde{a}_{nj}$	…	$\tilde{a}_{nn}$

4. NUMERICAL ILLUSTRATION

Let us consider an IFAP, where a farmer intends to plant four different crops in each of four equal sized paddocks. Rows representing four different crops Crop1, Crop2, Crop3 and Crop4 and Columns representing the four equal sized paddocks like P1, P2, P3 and P4. The nutrient requirements required for different crops vary and the paddocks vary in soil fertility. Thus the cost of the fertilizers which must be applied depends on which crop is grown in which paddock. Let [$\tilde{C}_{ij}$] be the cost matrix whose elements are given triangular Intuitionistic fuzzy numbers. The farmer's objective is to find the optimal assignment of paddocks to crops in such a manner that the total fertilizer cost becomes minimum.

		Paddocks			
		P1	**P2**	**P3**	**P4**
	Crops1	(7,9,1)(5,9,13)	(5,19,27)(1,19,32)	(10,23,30)(6,23,30)	(15,28,59)(11,28,59)
Crops	Crops2	(7,8,15)(1,8,21)	(6,8,12)(4,8,19)	(4,14,28)(3,14,31)	(3,14,26)(3,14,27)
	Crops3	(4,8,21)(1,8,24)	(2,10,24)(1,10,27)	(8,20,23)(5,20,26)	(7,9,11)(5,9,13)
	Crops4	(2,13,25)(1,13,26)	(7,11,24)(4,11,27)	(11,12,19)(5,12,25)	(10,23,30)(6,23,30)

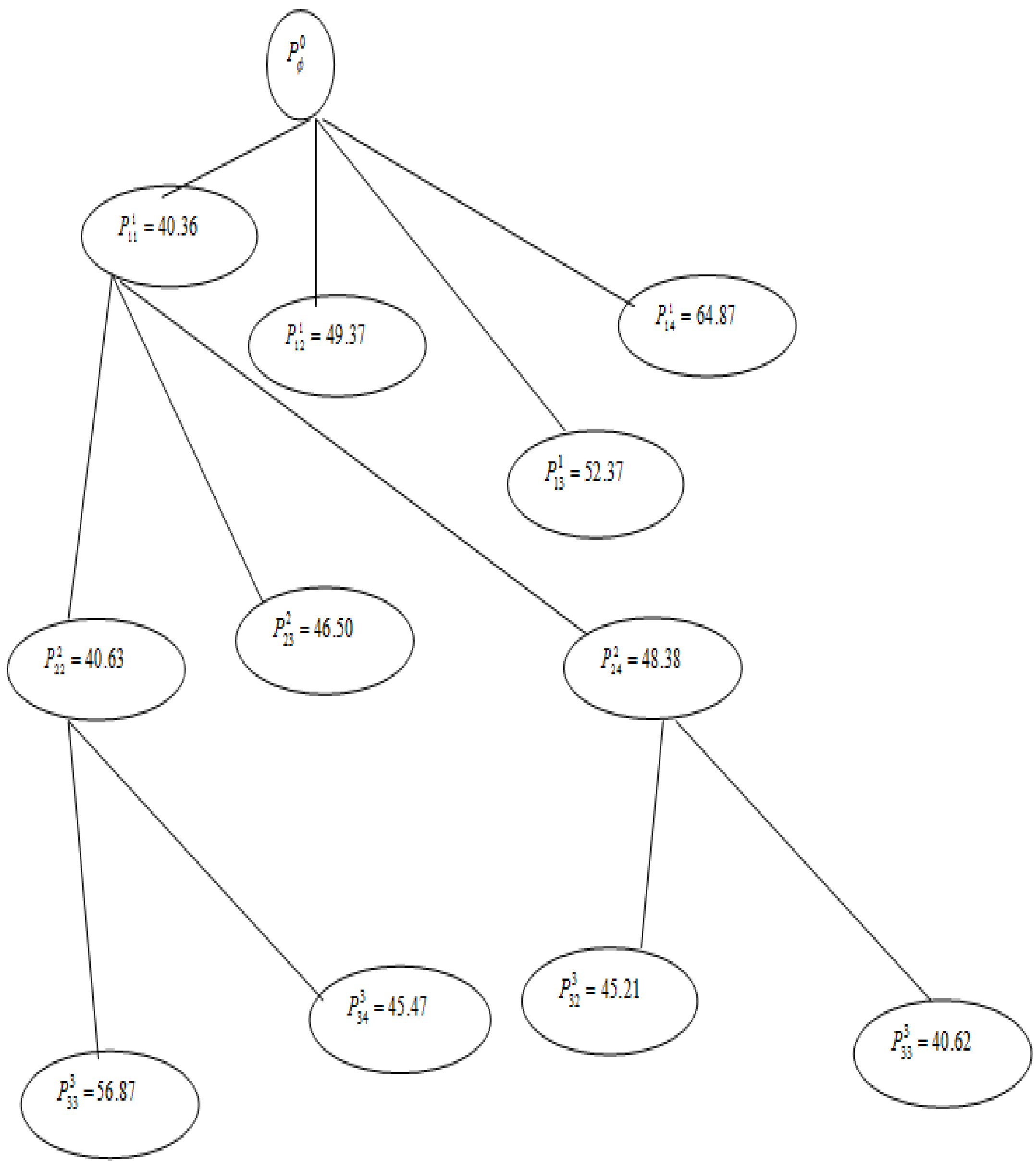

Fig. 1. Various nodes of tree using

The Accuracy function is

$$\mathrm{H}(\bar{a}^{I}) = \frac{(a_1 + 2a_2 + a_3) + (a_1^{'} + 2a_2 + a_3^{'})}{8}$$

$C_{11} = 9$, $C_{12} = 17.62$, $C_{13} = 21$, $C_{14} = 32$,

$C_{21} = 9.5$, $C_{22} = 9.12$, $C_{23} = 15.25$, $C_{24} = 14.37$

$C_{31} = 10.5$, $C_{32} = 11.75$, $C_{33} = 17.75$, $C_{34} = 9$,

$C_{41} = 13.25$, $C_{42} = 13.25$, $C_{43} = 13.50$, $C_{44} = 21$

Therefore, the cost matrix = $\begin{bmatrix} 9.00 & 17.62 & 21.00 & 32.00 \\ 9.50 & 9.12 & 15.25 & 14.37 \\ 10.50 & 11.75 & 17.75 & 09.00 \\ 13.25 & 13.25 & 13.50 & 21.00 \end{bmatrix}$

Sometimes we don't know the exact cost that occurs by assigning any crop to any paddock. It may be around in any number. Then in such situation we can apply this in agricultural fields. If cost occurs due to various factors and one factor may occurs the same cost, the costs are so formed interns of INFNs.

Now the optimal solution can be obtained from the following Assignment problem after replacing the corresponding values of of a_{ij} with the obtained cost matrix as shown below:

		Paddocks			
		P1	**P2**	**P3**	**P4**
Crops	Crops1	9.00	17.62	21.00	32.00
	Crops2	9.50	9.125	15.25	14.37
	Crops3	10.50	11.75	17.75	9.00
	Crops4	13.25	13.25	13.50	21.00

In order to get the optimal allocation, we solve the above problem using Branch and Bound Method. The branch and bound tree is shown in Fig. 1.

The conventional assignment problem in the Linear programming problem form can be obtained by replacing above values for their corresponding values. After solving, we get the optimal solution 40.62 and the optimal allocation is (1,1), (2,2), (3,4),and (4,3). It means that crop1 is assigned to P1 Paddock, crop2 is assigned to P2 Paddock, crop3 is assigned to P4 Paddock and crop4 is assigned to P3 Paddock, With optimal fertilizer cost 40.62.

5. CONCLUSION

In this paper, the assignment cost has been considered as an Intuitionistic fuzzy numbers. By defuzzifying with the accuracy function, the assignment costs are converted into crisp values and the optimum solution is obtained by using Branch and Bound method. This method can be utilized to all type of Intuitionistic fuzzy assignment problem and is easy to apply.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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