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Estimation of Stress-Strength Reliability for Exponentiated Inverted Weibull Distribution Based on Lower Record Values

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Abstract

The estimation of stress-strength reliability function based on record values has attracted the attention of authors because of its important role in industrial tests, where in some situations not all observations are considered in the study but only observations which are more extreme than the current observations, which represent record values, are the subject of the study. This paper deals with the estimation of $R = P[Y < X]$, when X and Y are two independently exponentiated inverted Weibull distributed random variables based on lower record values. Non-Bayesian estimator using maximum likelihood method and Bayesian estimators using squared error and LINEX loss functions are derived. The exact confidence interval of the reliability is also derived. For illustrative purposes, analysis of a simulated data set has been performed to compare the different estimators and to investigate the coverage probabilities of confidence intervals.

Keywords: Exponentiated inverted Weibull distribution; stress- strength reliability; lower record values.

1 Introduction

The inverted Weibull distribution is one of the most popular lifetime probability distributions which can be used and applied to a wide range of situations including applications in reliability engineering discipline, medicine and ecology. Some applications used the inverted Weibull distribution as a model of a variety of failure characteristics such as infant mortality, useful life and wear-out periods as mentioned by [1,2].

An exponentiated distribution is a generalization of the distribution through adding a new shape parameter by the exponentiation of the distribution function F in the form F^{θ}. Extending the inverted Weibull

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distribution to the exponentiated inverted Weibull distribution has been proposed by [3] through adding a new shape parameter which might be address the lack of fit of the inverted Weibull distribution for modeling lifetime data which indicate non-monotone failure rates.

According to this study, it is observed that the exponentiated inverted Weibull distribution (EIW) can be serving as alternative to the inverted Weibull distribution and it is expected that in some situations it might work better than the inverted Weibull distribution. For $\theta > 0$ and > 0, the probability density function (pdf) and cumulative distribution function (cdf) of the EIW (θ, β) are given, respectively, as follows:

$$f(x;\beta,\theta) = \theta\beta x^{-(\beta+1)}(exp - x^{-\beta})^{\theta}, x > 0, \tag{1}$$

$$F(x;\beta,\theta) = (exp - x^{-\beta})^{\theta}. \tag{2}$$

Record values have a great important role in real life problems involving data relating to several fields such as weather, economics and sports data. The statistical study of record values began with [4] who introduced the main idea of record values, record times, inter record times and formulated the theory of record values as a model for successive extremes in a sequence of independently and identically distributed random variables. The prominent theoretical contributions and inference issues of the record values have been proposed by [5-8]. The record values can be classified into the lower and the upper records. An observation X_i will be called a lower record values if its values are less than all previous observation (i.e., $X_i < X_j$ for every $i > j$) and it will be called an upper record values if its values exceeds that of all previous observations (i.e., $X_i > X_j$ for every $i < j$).

In reliability stress- strength model, the system or the component is still working as far as the stress doesn't exceed strength. The probability of this event is the stress-strength reliability model, which includes two random variables X and Y where X denotes the strength of the system or the component while Y denotes the stress which is subjected to it. The probability $R = P[Y < X]$ is the stress- strength reliability function. The stress-strength model is of special importance in reliability literature because it has an important role and useful applications in various fields.

Due to the practical point of views of reliability stress-strength model, the estimation problem of $R = P[Y < X]$ has attracted the attention of many authors. This model was introduced by [9] and the main development was considered by [10]. The estimation problem of R has been investigated in the literature for many distributions by several authors; see for example [11-14].

Recently, the growing interest about the estimation of stress-strength reliability R associated with record values have been raised in many fields such as industrial test. The estimation of stress strength R based on record values is considered by [15] for generalized exponential distribution. Subsequent papers extended this work for some lifetime models, for instance [16-18], for one and two parameters exponential distribution, [19] for type I generalized logistic distribution, [20] for two-parameter Weibull distribution, [21] for inverse Rayleigh distribution, [22] for exponentiated Weibull distribution.

This article aims to estimate the stress-strength reliability function $R = P[Y < X]$ when the strength X and the stress Y are two independent variables of EIW distribution and their measurements are in terms of lower record values. Assuming that the scale parameter is known, maximum likelihood estimate and exact confidence interval of R are derived. In addition, the Bayes estimate of R based on independent gamma priors for the unknown parameters are obtained under squared error and LINEX loss functions. The procedures are illustrated by analyzing a simulated data. The rest of the paper is organized as follows. In Section (2) maximum likelihood estimator and exact confidence interval of R are discussed. In Section (3), the Bayes estimates of R against both squared error and LINEX loss functions are discussed. Steps of simulation study are proposed in Section (4). Finally, conclusions appear in Section (5).

2 Likelihood Inference

In this section maximum likelihood estimate (MLE) of R is obtained. Also, the exact confidence interval of R is derived.

2.1 MLE of the Reliability Function *R*

Let X be the strength of a system or component which is subjected to the stress Y. Assuming that $X \sim \text{EIW}(\theta_1, \beta)$, and $Y \sim \text{EIW}(\theta_2, \beta)$, then the reliability function is obtained as follows;

$$R = P(Y < X) = \int_0^\infty P[Y < X | Y = y] f_Y(y) dy,$$

$$R = \int_0^\infty \theta_1 \beta x^{-(\beta+1)} (exp - x^{-\beta})^{\theta_1} (exp - x^{-\beta})^{\theta_2} dx = \frac{\theta_1}{\theta_1 + \theta_2}. \tag{3}$$

Let $\underline{r} = (r_0, r_1, \ldots, r_n)$ be a set of the first observed lower record values of size $(n+1)$ from EIW with parameters (θ_1, β) and $\underline{s} = (s_0, s_1, \ldots, s_m)$ be an independent set of the observed first lower record values of size $(m+1)$ from EIW with parameters (θ_2, β) where β assumed known. The likelihood functions for both observed $\underline{r}$ and $\underline{s}$ are given, respectively, (See [7]), by

$$L_1(\theta_1, \beta | \underline{r}) = f(r_n) \prod_{i=0}^{n-1} \frac{f(r_i)}{F(r_i)}; \;\; 0 < r_n < r_{n-1} < \cdots < r_0 < \infty, \tag{4}$$

and,

$$L_2(\theta_2, \beta | \underline{s}) = g(s_m) \prod_{j=0}^{m-1} \frac{g(s_i)}{G(s_i)}; \;\; 0 < s_m < s_{m-1} < \cdots < s_0 < \infty, \tag{5}$$

where $f(.)$ and $F(.)$ are respectively, the pdf and cdf of X and $g(.)$ and $G(.)$ are the pdf and the cdf of Y respectively. The likelihood function of the observed record values $\underline{r}$ and $\underline{s}$ are obtained, as follows

$$L_1(\theta_1, \beta | \underline{r}) = (\theta_1 \beta)^{n+1} e^{-\theta_1 (r_n)^{-\beta}} \prod_{i=0}^{n} r_i^{-(\beta+1)}, \tag{6}$$

and,

$$L_2(\theta_2, \beta | \underline{s}) = (\theta_2 \beta)^{m+1} e^{-\theta_2 (s_m)^{-\beta}} \prod_{j=0}^{m} s_j^{-(\beta+1)}. \tag{7}$$

Therefore, the joint log- likelihood function of the observed $\underline{r}$ and $\underline{s}$ denoted by l takes the following form

$$\begin{aligned} l = (n+1) ln\theta_1 + (m+1) ln\theta_2 + (n+m+2) ln\beta - \theta_1 r_n^{-\beta} - \theta_2 s_m^{-\beta} \\ -(\beta+1)\left[\sum_{i=0}^{n} \ln(r_i) + \sum_{j=0}^{m} \ln(s_j)\right] \end{aligned} \tag{8}$$

The maximum likelihood estimators of θ_1 and θ_2, denoted by $\hat{\theta}_1$ and $\hat{\theta}_2$, based on the observed lower record are obtained by solving the following equations

$$\frac{\partial l}{\partial \theta_1} = \frac{(n+1)}{\hat{\theta}_1} - r_n^{-\beta} = 0, \tag{9}$$

$$\frac{\partial l}{\partial \theta_2} = \frac{(m+1)}{\hat{\theta}_2} - s_m{}^{-\beta} = 0. \qquad (10)$$

From (9) and (10), $\hat{\theta}_1$ and $\hat{\theta}_2$ are obtained as follows

$$\hat{\theta}_1 = \frac{(n+1)}{r_n{}^{-\beta}}, \hat{\theta}_2 = \frac{(m+1)}{s_m{}^{-\beta}} \qquad (11)$$

Hence, the maximum likelihood estimator of of R, denoted by $\hat{R}$, is given by substitute $\hat{\theta}_1$ and $\hat{\theta}_2$ in (3) as follows

$$\hat{R} = \frac{\hat{\theta}_1}{\hat{\theta}_1 + \hat{\theta}_2}. \qquad (12)$$

2.2 Confidence Interval of *R*

In this subsection the exact confidence interval of R is derived. Therefore the distribution of R must be obtained. To derive the distribution of of R firstly the distributions of $\hat{\theta}_1$ and $\hat{\theta}_2$must be obtained. Consider that $\hat{\theta}_1 = \frac{(n+1)}{r_n{}^{-\beta}}$, then according to [7], the pdf of R_n is given by

$$f_{R_n}(r_n) = \frac{1}{\Gamma(n+1)}[-\ln F(r_n)]^n f(r_n) \quad , \qquad 0 < r_n < \infty, n = 0,1,2, \ldots$$

$$f_{R_n}(r_n) = \frac{\theta_1{}^{n+1}\beta}{\Gamma(n+1)} r_n{}^{-\beta(n+1)-1} e^{-\theta_1 r_n{}^{-\beta}}.$$

Similarly, for $\hat{\theta}_2 = \frac{(m+1)}{s_m{}^{-\beta}}$, the pdf of $\mathrm{S_m}$ is given by

$$g_{\mathrm{S_m}}(s_m) = \frac{\theta_2{}^{m+1}\beta}{\Gamma(m+1)} s_m{}^{-\beta(m+1)-1} e^{-\theta_2 s_m{}^{-\beta}}, 0 < s_m < \infty, \; m = 0,1,2, \ldots.$$

Therefore, the probability density functions of $\hat{\theta}_1 = \frac{(n+1)}{r_n{}^{-\beta}}$ and $\hat{\theta}_2 = \frac{(m+1)}{s_m{}^{-\beta}}$, are obtained as follows;

Let $Z_1 = \hat{\theta}_1 = \frac{(n+1)}{r_n{}^{-\beta}}$, $Z_2 = \hat{\theta}_2 = \frac{(m+1)}{s_m{}^{-\beta}}$, it is easy to show that the probability density functions of Z_1 and Z_2 are given as follows

$$f(z_1) = \frac{[(n+1)\theta_1]^{n+1}}{\Gamma(n+1) z_1{}^{n+2}} e^{\frac{-(n+1)\theta_1}{z_1}}, z_1 > 0, \;\; f(z_2) = \frac{[(m+1)\theta_2]^{m+1}}{\Gamma(m+1) z_2{}^{m+2}} e^{\frac{-(m+1)\theta_2}{z_2}}, z_2 > 0.$$

These are recognized as the inverted gamma distribution; that is, Z_1 has inverted gamma $[(n+1),(n+$ *1θ1*, and similarly *Z2* has inverted gamma *m+1,m+1θ2*, therefore the pdf of *R* can be obtained as follows

$$\hat{R} = \frac{Z_1}{Z_1 + Z_2} = \frac{1}{1 + \frac{Z_2}{Z_1}}.$$

Considering, Z_2/Z_1 , it is easy through the properties of the inverted gamma distribution and its relation with the gamma distribution to show that

$$\frac{2(n+1)\theta_1}{Z_1} \sim \chi^2_{2(n+1)} \quad \text{and} \quad \frac{2(m+1)\theta_2}{Z_2} \sim \chi^2_{2(m+1)} \tag{13}$$

Since, Z_1 and Z_2 are independent, then it can be shown that $\frac{Z_2}{Z_1} \sim \frac{\theta_2}{\theta_1} F_{2(n+1),2(m+1)}$, where $F_{2(n+1),2(m+1)}$ is F distribution with $2(n+1), 2(m+1)$ degrees of freedom. The exact distribution of $\hat{R}$ written as

$$f(\hat{R}) = \frac{1}{1+\frac{\theta_2}{\theta_1} F_{2(n+1),2(m+1)}}.$$

A $(1-\alpha)\%$ confidence interval for R, based on lower record values is (L_1, U_1), where

$$L_1 = \left[1 + \frac{Z_2}{Z_1 F_{\alpha/2,2(n+1),2(m+1)}}\right]^{-1}, U_1 = \left[1 + \frac{Z_2}{Z_1 F_{1-\alpha/2,2(n+1),2(m+1)}}\right]^{-1}, \tag{14}$$

are the lower and upper *α/2-th* percentile points of $F_{2(n+1),2(m+1)}$.

3 Bayesian Inference

In this section, the Bayes estimate of R is obtained under the assumption that the shape parameters θ_1 and θ_2 are random variables for both populations. Two different loss functions are used; squared error and LINEX.

3.1 Bayes Estimate of *R* Based on Squared Error Loss Function

To obtain the Bayes estimate of R under squared error loss function, firstly it must to obtain the Bayes estimate of θ_1 and θ_2. Then according to [23], the conjugate priors of θ_1 and θ_2 are selected to be gamma distributions as follows

$$\pi_1(\theta_1) \propto {\theta_1}^{a-1} e^{-b\theta_1} \quad \text{and} \quad \pi_2(\theta_2) \propto {\theta_2}^{c-1} e^{-d\theta_2},$$

where a, b, c and d are the parameters of prior distributions of θ_1 and θ_2 respectively.

The posterior distributions of θ_1 and θ_2, denoted by $\pi_1^*(\theta_1)$ and $\pi_2^*(\theta_2)$, are obtained by combining the likelihood functions (6) and (7) and the prior density functions $\pi_1(\theta_1)$, $\pi_2(\theta_2)$ as follows

$$\pi_1^*(\theta_1) = \frac{[{r_n}^{-(\beta)} + b]^{n+a+1}}{\Gamma(n+a+1)} {\theta_1}^{n+a} e^{-\theta_1[b+{r_n}^{-\beta}]}, \tag{15}$$

$$\pi_2^*(\theta_2) = \frac{[{s_m}^{-(\beta)} + d]^{m+c+1}}{\Gamma(m+c+1)} {\theta_2}^{m+c} e^{-\theta_1[d+{s_m}^{-\beta}]}. \tag{16}$$

The Bayes estimates of θ_1 and θ_2 under squared error loss function, denoted by $\hat{\theta}_{1(SE)}$ and $\hat{\theta}_{2(SE)}$, are the posterior means which can be obtained as follows

$$\hat{\theta}_{1(SE)} = \int_0^\infty \theta_1 \, \pi_1^*(\theta_1) d\theta_1 = \frac{n+a+1}{[{r_n}^{-(\beta)}+b]},$$

and,

$$\hat{\theta}_{2(SE)} = \int_0^{\infty} \theta_2 \, \pi_2^*(\theta_2) d\theta_2 = \frac{m+c+1}{[s_m^{-(\beta)}+d]} .$$

Therefore the Bayes estimate of R under squared error loss function, denoted by $\hat{R}_{SE}$ can be obtained by substitute $\hat{\theta}_{1(SE)}$ and $\hat{\theta}_{2(SE)}$ in Equation (3) as follows

$$\hat{R}_{SE} = \frac{\hat{\theta}_{1(SE)}}{\hat{\theta}_{1(SE)} + \hat{\theta}_{2(SE)}}.$$

3.2 Bayes Estimate of *R* Based on LINEX Loss Function

In this subsection the Bayes estimate of R under LINEX loss function is obtained. The Bayes estimate of θ_1 and θ_2 under LINEX loss function, denoted by $\hat{\theta}_{1(LINEX)}$ and $\hat{\theta}_{2(LINEX)}$, are obtained as follows

$$\hat{\theta}_{1(LINEX)} = \frac{-1}{h} \ln\, [E(e^{-h\theta_1})] = \frac{-1}{h} ln \left[\frac{[r_n^{-(\beta)} + b]^{(n+a+1)}}{[r_n^{-(\beta)} + b + h]^{(n+a+1)}} \right],$$

$$\hat{\theta}_{2(LINEX)} = \frac{-1}{h} \ln\, [E(e^{-h\theta_2})] = \frac{-1}{h} ln \left[\frac{[s_m^{-(\beta)} + d]^{(m+c+1)}}{[s_m^{-(\beta)} + d + h]^{(m+c+1)}} \right].$$

Therefore the Bayes estimate of R under LINEX loss function, denoted by $\hat{R}_{LINEX}$ can be obtained by substitute $\hat{\theta}_{1(LINEX)}$ and $\hat{\theta}_{2(LINEX)}$ in Equation (3) as follows

$$\hat{R}_{LINEX} = \frac{\hat{\theta}_{1(LINEX)}}{\hat{\theta}_{1(LINEX)} + \hat{\theta}_{2(LINEX)}}.$$

3.3 Bayes Confidence Interval of *R*

In Bayesian inference, to derive the distribution of stress-strength function R, the posterior distribution of θ_1 and θ_2 must be considered. According to Equations (15 and 16), the posterior distribution of θ_1 and θ_2 has gamma distribution as follows:

$$\pi_1^*(\theta_1|r) \sim Gamma[n+a+1, (r_n^{-(\beta)} + b)], \quad (17)$$

$$\pi_2^*(\theta_2|s) \sim Gamma[m+c+1, (s_m^{-(\beta)} + d)]. \quad (18)$$

From the properties of gamma distribution and its relation with chi-square distribution, it is easy to show that

$$2(r_n^{-(\beta)} + b)[\theta_1|r] \sim \chi^2_{2(n+a+1)} \text{ and } 2(s_m^{-(\beta)} + d)[\theta_2|s] \sim \chi^2_{2(m+c+1)}. \quad (19)$$

Then, the posterior distribution of reliability function R, denoted by $\hat{R}_1$ is

$$f(\hat{R}_1) = \left[1 + \frac{(s_m^{-(\beta)} + d)(n+a+1)}{(r_n^{-(\beta)} + b)(m+c+1)} \mathrm{F}_{2(m+c+1),2(n+a+1)} \right]^{-1}.$$

Therefore, a Bayesian$(1-\alpha)\%$ confidence interval for R based on lower record values is (L_2, U_2), where

$$L_2 = \left[1 + \frac{\left(s_m^{-(\beta)} + d\right)(n+a+1)}{(r_n^{-(\beta)} + b)(m+c+1)} F_{1-\alpha/2, 2(m+c+1), 2(n+a+1)}\right]^{-1},$$

and,

$$U_2 = \left[1 + \frac{\left(s_m^{-(\beta)} + d\right)(n+a+1)}{(r_n^{-(\beta)} + b)(m+c+1)} F_{\alpha/2, 2(m+c+1), 2(n+a+1)}\right]^{-1}, \tag{20}$$

are the lower and upper *α/2-th* percentile points of $F_{2(m+c+1), 2(n+a+1)}$.

4 Simulation Study

In this section a simulation study is designed to investigate and to compare the performance of MLE and Bayes estimates (under squared error and LINEX loss functions). The exact values of stress-strength reliability R are chosen as R= 0.25, 0.54 and 0.75. The estimates of stress-strength reliability R through maximum likelihood and Bayesian techniques are calculated. The exact confidence interval of R is derived for both methods. The simulation study is designed through the following steps:

1. Generate 5000 uniform (0, 1) random variables and then get the corresponding EIW random samples of sample size 200 through the transformation technique.
2. Select from each vector the first $(n+1), n = 2(1)9,$ lower record values $r_0, r_1, \dots, r_n$ for the values of strength random variables X under the assumption that β is known.
3. Repeat the previous two steps to generate 5000 random samples of size 200 from EIW and select from each vector the first $(m+1)$, $m = 2(1)9$ lower record values $s_0, s_1, \dots, s_m$ for the values of stress random variables Y under the assumption that β is known.
4. The MLE of $\hat{\theta}_1$ and $\hat{\theta}_2$ are obtained from (11), then the MLE of R is obtained by substitute $\hat{\theta}_1$ and $\hat{\theta}_2$ in (12). The exact confidence intervals of R using (14) are constructed with confidence level at $\alpha = 0.05$.
5. For given values of prior parameters $a = c = 2$, $b = d = 3$, the Bayes estimates of R under squared error and LINEX loss functions are obtained. Also, the 95% Bayes confidence interval of stress-strength reliability under both loss functions are calculated.
6. Compute the average for R, $\hat{R}_{SE}$and $\hat{R}_{LINEX}$, mean square errors (MSEs), coverage percentages, and average probability interval lengths.

5 Results and Discussion

Simulation results are reported in Tables (1-6) and represented through Figs. (1-3). From these tables, the following results can be observed on the properties of reliability estimates

1. The coverage percentage of MLE is better than that of the Bayesian estimator at $R = 0.25$ according to Tables (1, 2).
2. Tables (1, 2) show that the coverage percentage of Bayesian estimator under LINEX loss function, where $h = 2$, is better than the Bayes one under square error loss function.
3. The coverage percentages of MLE and Bayes estimate are almost equal at $R = 0.54$, 0.75 according to Tables (3-6).
4. The average length of the confidence intervals for MLE is shorter than the Bayes one according to Tables (1-6).
5. For fixed value of n, the coverage percentages increase when $n = m$.
6. It is observed from Tables (1-6) that when n and m increase, the coverage percentages decrease for different estimators at different values of R.

7. The length of the non-Bayes confidence interval for the reliability R is shorter than the Bayes confidence interval (under both loss functions).

Table 1. Simulation results for MLEs and Bayes estimates under squared error loss function when $\theta_1 = 1, \theta_2 = 3$ and β=1 with R-exact=0.25

n	m	MLE				Bayes (squared loss function)			
		Mean	**MSE**	**Length**	**Coverage**	**Mean**	**MSE**	**Length**	**Coverage**
2	2	0.219	0.069	0.356	0.999	0.363	0.032	1.782	0.980
3	2	0.146	0.050	0.290	0.968	0.312	0.027	1.586	0.930
	3	0.184	0.048	0.315	0.999	0.303	0.023	1.527	0.929
	4	0.182	0.048	0.290	0.910	0.264	0.025	1.27	0.832
4	3	0.133	0.041	0.256	0.920	0.251	0.022	1.362	0.807
	4	0.158	0.038	0.275	0.988	0.257	0.019	1.334	0.812
	5	0.137	0.038	0.230	0.831	0.208	0.024	1.024	0.658
5	4	0.116	0.037	0.217	0.844	0.205	0.022	1.127	0.631
	5	0.144	0.030	0.253	0.997	0.232	0.015	1.215	0.718
	6	0.106	0.037	0.180	0.748	0.165	0.026	0.834	0.505
6	5	0.101	0.037	0.180	0.745	0.168	0.025	0.907	0.488
	6	0.134	0.028	0.228	0.994	0.213	0.013	1.108	0.609
	7	0.086	0.039	0.144	0.667	0.135	0.029	0.685	0.385
7	6	0.085	0.038	0.146	0.647	0.138	0.029	0.725	0.367
	7	0.126	0.026	0.207	0.977	0.198	0.013	1.009	0.521
	8	0.071	0.041	0.116	0.594	0.112	0.032	0.568	0.284
8	8	0.121	0.025	0.193	0.996	0.190	0.012	0.961	0.421
9	9	0.113	0.027	0.171	0.908	0.174	0.015	0.825	0.348

Table 2. Simulation results for Bayes estimates under LINEX loss function when $\theta_1 = 1, \theta_2 = 3$ and β=1 with R-exact =0.25

n	m	Bayes (LINEX loss function) , $h = 2$				Bayes (LINEX loss function), $h = -2$			
		Mean	**MSE**	**length**	**Coverage**	**Mean**	**MSE**	**length**	**Coverage**
2	2	0.378	0.032	1.782	0.988	0.334	0.032	1.782	0.952
	2	0.334	0.029	1.586	0.946	0.272	0.027	1.586	0.872
3	3	0.321	0.023	1.527	0.953	0.273	0.025	1.527	0.851
	4	0.275	0.023	1.270	0.853	0.244	0.028	1.270	0.755
	3	0.271	0.022	1.362	0.855	0.217	0.024	1.362	0.691
4	4	0.275	0.018	1.334	0.876	0.230	0.021	1.334	0.695
	5	0.220	0.023	1.024	0.707	0.189	0.027	1.024	0.577
	4	0.222	0.022	1.127	0.714	0.178	0.025	1.127	0.505
5	5	0.250	0.013	1.215	0.794	0.207	0.017	1.215	0.585
	6	0.176	0.025	0.834	0.560	0.149	0.028	0.834	0.424
	5	0.182	0.025	0.907	0.554	0.147	0.028	0.907	0.375
6	6	0.230	0.012	1.108	0.692	0.189	0.016	1.108	0.478
	7	0.145	0.028	0.685	0.440	0.121	0.031	0.685	0.304
	6	0.150	0.028	0.725	0.429	0.121	0.031	0.725	0.279
7	7	0.215	0.011	1.009	0.616	0.176	0.016	1.009	0.380
	8	0.121	0.031	0.568	0.339	0.100	0.034	0.568	0.210
8	8	0.206	0.009	0.961	0.517	0.169	0.015	0.961	0.320
9	9	0.188	0.014	0.825	0.433	0.155	0.018	0.825	0.257

Table 3. Simulation results for MLEs and Bayes estimates under squared error loss function when $\theta_1 = 3, \theta_2 = 2.5$ and β=1 with R-exact =0.545

n	m	MLE				Bayes (squared loss function)			
		Mean	MSE	Length	Coverage	Mean	MSE	Length	Coverage
2	2	0.555	0.106	0.465	0.999	0.514	0.007	2.175	0.997
3	2	0.465	0.106	0.453	0.968	0.522	0.017	1.841	0.965
	3	0.561	0.081	0.460	0.999	0.523	0.010	1.769	0.999
	4	0.551	0.092	0.410	0.910	0.466	0.037	1.475	0.910
4	3	0.478	0.093	0.422	0.920	0.496	0.033	1.528	0.920
	4	0.557	0.068	0.441	0.988	0.522	0.015	1.499	0.988
	5	0.492	0.098	0.366	0.831	0.433	0.060	1.155	0.831
5	4	0.461	0.095	0.376	0.844	0.459	0.055	1.242	0.844
	5	0.572	0.052	0.433	0.997	0.534	0.012	1.343	0.997
	6	0.432	0.109	0.321	0.748	0.391	0.083	0.925	0.748
6	5	0.424	0.110	0.319	0.745	0.411	0.083	0.990	0.745
	6	0.570	0.044	0.416	0.994	0.535	0.012	1.213	0.994
	7	0.382	0.125	0.276	0.667	0.350	0.106	0.752	0.667
7	6	0.376	0.130	0.266	0.647	0.360	0.111	0.786	0.647
	7	0.565	0.041	0.394	0.977	0.530	0.017	1.097	0.977
	8	0.337	0.140	0.238	0.594	0.312	0.127	0.620	0.594
8	8	0.572	0.033	0.386	0.996	0.538	0.011	1.040	0.996
9	9	0.534	0.054	0.336	0.908	0.499	0.036	0.892	0.908

Table 4. Simulation results for Bayes estimates under LINEX loss function when $\theta_1 = 3, \theta_2 = 2.5$ and β=1 with R-exact =0.545

n	m	Bayes (LINEX loss function), $h = 2$				Bayes (LINEX loss function), $h = -2$			
		Mean	MSE	Length	Coverage	Mean	MSE	Length	Coverage
2	2	0.512	0.005	2.175	0.997	0.519	0.012	2.175	0.997
3	2	0.525	0.015	1.841	0.965	0.513	0.023	1.841	0.965
	3	0.519	0.007	1.769	0.999	0.530	0.016	1.769	0.999
	4	0.458	0.035	1.475	0.910	0.483	0.042	1.475	0.910
4	3	0.498	0.031	1.528	0.920	0.492	0.039	1.528	0.920
	4	0.518	0.012	1.499	0.988	0.529	0.021	1.499	0.988
	5	0.426	0.058	1.155	0.831	0.447	0.064	1.155	0.831
5	4	0.459	0.053	1.242	0.844	0.458	0.060	1.242	0.844
	5	0.530	0.009	1.343	0.997	0.543	0.017	1.343	0.997
	6	0.386	0.082	0.925	0.748	0.401	0.087	0.925	0.748
6	5	0.409	0.082	0.990	0.745	0.413	0.088	0.990	0.745
	6	0.531	0.010	1.213	0.994	0.544	0.017	1.213	0.993
	7	0.346	0.105	0.752	0.667	0.358	0.109	0.752	0.667
7	6	0.358	0.110	0.786	0.647	0.363	0.115	0.786	0.647
	7	0.525	0.015	1.097	0.977	0.538	0.021	1.097	0.977
	8	0.308	0.126	0.620	0.594	0.318	0.129	0.620	0.594
8	8	0.533	0.008	1.040	0.996	0.546	0.015	1.040	0.995
9	9	0.494	0.034	0.892	0.908	0.508	0.039	0.892	0.908

Table 5. Simulation results for MLEs and Bayes estimates under squared error loss function when $\theta_1 = 3, \theta_2 = 1$ and β=1 with R-exact =0.75

n	m	MLE				Bayes (squared loss function)			
		Mean	MSE	Length	Coverage	Mean	MSE	Length	Coverage
2	2	0.784	0.069	0.351	0.999	0.638	0.032	2.43	0.999
3	2	0.718	0.088	0.359	0.968	0.637	0.044	2.057	0.967
	3	0.819	0.046	0.316	0.999	0.697	0.023	1.921	0.999
	4	0.78	0.085	0.254	0.910	0.661	0.067	1.581	0.910
4	3	0.736	0.086	0.296	0.920	0.653	0.064	1.654	0.920
	4	0.829	0.042	0.280	0.988	0.728	0.025	1.603	0.988
	5	0.717	0.121	0.215	0.831	0.63	0.107	1.227	0.831
5	4	0.705	0.115	0.236	0.844	0.632	0.101	1.323	0.844
	5	0.855	0.031	0.251	0.997	0.766	0.016	1.421	0.997
	6	0.647	0.162	0.182	0.748	0.579	0.151	0.977	0.748
6	5	0.641	0.164	0.180	0.745	0.581	0.153	1.044	0.745
	6	0.86	0.030	0.230	0.994	0.78	0.016	1.277	0.994
	7	0.579	0.204	0.151	0.667	0.525	0.195	0.791	0.667
7	6	0.565	0.215	0.140	0.647	0.516	0.206	0.825	0.647
	7	0.854	0.037	0.206	0.977	0.780	0.024	1.151	0.977
	8	0.517	0.243	0.126	0.594	0.472	0.235	0.650	0.594
8	8	0.870	0.026	0.199	0.996	0.801	0.013	1.089	0.996
9	9	0.801	0.074	0.163	0.908	0.740	0.062	0.932	0.908

Table 6. Simulation results for Bayes estimates under LINEX loss function when $\theta_1 = 3, \theta_2 = 1$ and β=1 with R-exact =0.75

n	m	Bayes (LINEX loss function), $h = 2$				Bayes (LINEX loss function), $h = -2$			
		Mean	MSE	Length	Coverage	Mean	MSE	Length	Coverage
2	2	0.623	0.033	2.430	0.999	0.667	0.032	2.430	0.999
3	2	0.628	0.043	2.057	0.967	0.653	0.048	2.057	0.967
	3	0.679	0.023	1.921	0.999	0.727	0.025	1.921	0.999
	4	0.640	0.068	1.581	0.910	0.695	0.068	1.581	0.910
4	3	0.641	0.063	1.654	0.920	0.673	0.067	1.654	0.920
	4	0.711	0.024	1.603	0.988	0.756	0.027	1.603	0.988
	5	0.613	0.106	1.227	0.831	0.756	0.109	1.227	0.831
5	4	0.620	0.100	1.323	0.844	0.651	0.103	1.323	0.844
	5	0.749	0.015	1.421	0.997	0.792	0.018	1.421	0.997
	6	0.565	0.150	0.977	0.748	0.601	0.153	0.977	0.748
6	5	0.569	0.152	1.044	0.745	0.597	0.155	1.044	0.745
	6	0.763	0.014	1.277	0.994	0.804	0.019	1.277	0.994
	7	0.513	0.194	0.791	0.667	0.542	0.197	0.791	0.667
7	6	0.506	0.205	0.825	0.647	0.530	0.208	0.825	0.647
	7	0.764	0.022	1.151	0.977	0.803	0.027	1.151	0.977
	8	0.462	0.234	0.650	0.594	0.486	0.236	0.650	0.594
8	8	0.785	0.011	1.089	0.996	0.822	0.016	1.089	0.996
9	9	0.726	0.060	0.932	0.908	0.760	0.065	0.932	0.908

Figs. (1-3) show the trend of mean square errors of the MLE and Bayesian estimators under square error and LINEX loss functions for the exact values of $R = 0.25$, 0.54 and 0.75, the following results can be observed on the mean square errors of the reliability estimates.

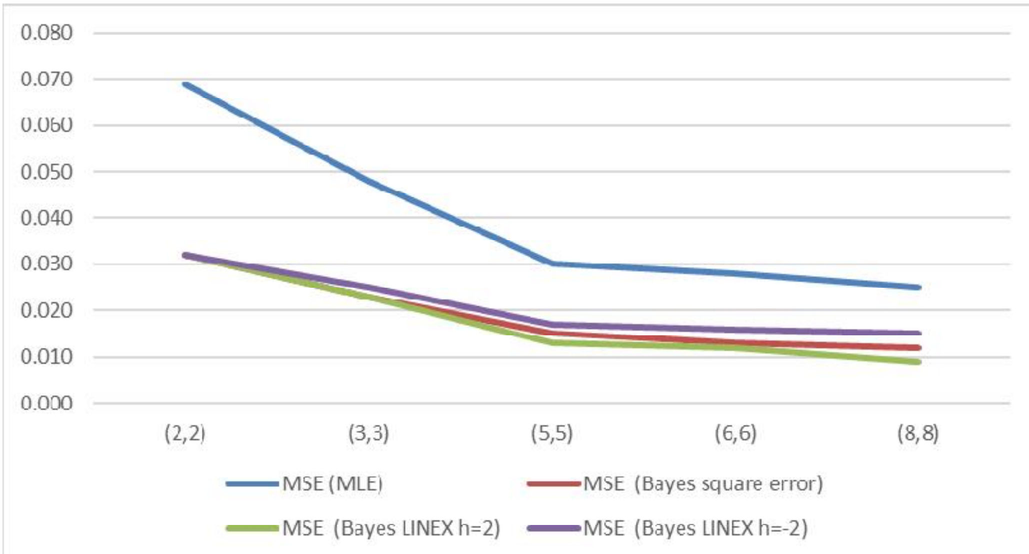

Fig. 1. MSE at $R = 0.25$ for Bayesian and Non-Bayesian estimator when $n = m$

Fig. 1 shows that the MSEs of the MLE and Bayesian estimator under squared error and LINEX loss functions decrease as n and m increase for the exact values of $R = 0.25$. Also, MSEs of Bayes estimate under LINEX loss function for $h = 2$ are the smallest one.

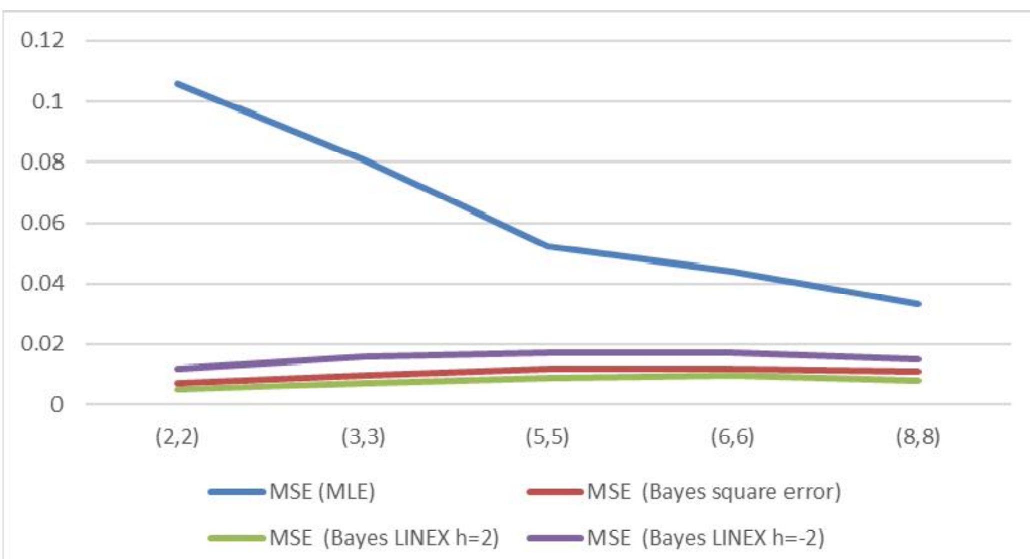

Fig. 2. MSE at $R = 0.54$ for Bayesian and Non-Bayesian estimator when $n = m$

It is clear from Fig. 2 that the MSEs of the Bayes estimator under square error and LINEX loss functions are less than the MSEs of MLE when $R = 0.54$. Also, Bayesian estimator under LINEX loss function for $h = 2$ has minimum MSEs.

The following observations can made from Fig. 3 as follows:

1. The MSEs of the MLE and Bayesian estimator under squared error and LINEX loss functions decrease as n and m increase for the exact values of $R = 0.75$.
2. The MSEs of the Bayes estimator under square error and LINEX loss functions are less than the MSEs of MLE when $R = 0.75$.
3. The MSEs of the Bayesian estimator under LINEX loss function for $h = 2$ are the smallest one corresponding to the MSEs for the other estimators.

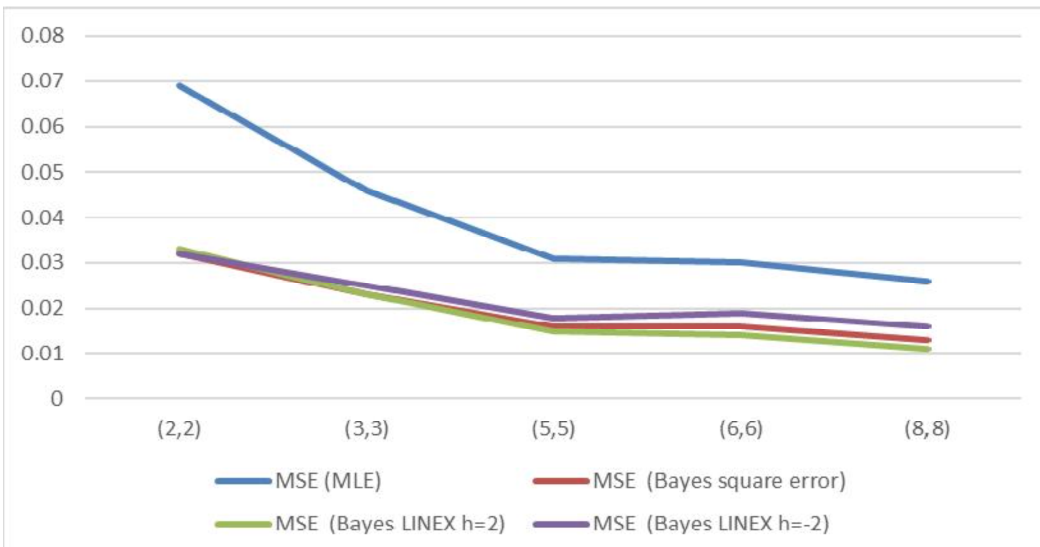

Fig. 3. MSE at $R = 0.75$ for Bayesian and Non-Bayesian estimator when $n = m$

6 Conclusion

In this paper, the MLE and Bayesian estimators are derived for the stress-strength reliability function R when the stress and strength variables are independently EIW distributions based on lower record values. Also, the exact Bayesian and non-Bayesian confidence interval of R, are derived.

Generally, the coverage percentage of MLE is better than coverage percentage of the Bayes estimator at $R = 0.25$. Also average confidence interval lengths of the MLE are shorter than the corresponding average confidence interval lengths of the Bayes estimators. The MSEs of the Bayesian estimator under LINEX loss function are less than the MSEs of MLE at different exact values of stress-strength reliability function R.

Regarding, the number of records (n and m) for the stress and strength variables, it is observed that for small values of n and m the coverage percentage increases. While the coverage percentage decreases as n and m increase.

Based on above the MLE is better than the Bayes estimates (under squared error and LINEX loss functions) in terms of the lengths of confidence intervals but the Bayes estimator under LINEX loss function is better than the MLE in terms of MSEs. Although the coverage percentages of maximum likelihood and Bayesian estimates are almost equal for R = 0.54, 0.75 but the lengths of confidence interval of MLE are shorter than the corresponding Bayes one. So, the MLE is better than the Bayes one in terms of the coverage percentage.

Competing Interests

Authors have declared that no competing interests exist.

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