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Investigation into Grade XI Students' Misconception about the Limit Concept: A Case Study at Samtse Higher Secondary School in Bhutan

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Author's contribution

The sole author designed, analysed, interpreted and prepared the manuscript.

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ABSTRACT

In the study of calculus, the concept of limit of function occupies a central role as it is important instruments used in the study of the theory of rate of change, continuity, integral calculus, and differential calculus. Despite its significance, the secondary students hold the inadequate understanding of the limit concepts, more over their concept image of the limit function deviated from the concept definition resulting in the misconception. This study aims to identify the misconception in the limit of function and possible causes of misconceptions. This study was done in two phases, a concept test based on limit of function was administered to all 25 students of Samtse Higher Secondary School. Subsequently, based on the errors and misconception demonstrated by students from the concept test, five students were purposively selected and interviewed to corroborate the finding from concept test to confirm the existence of misconception and its causes. Data from the transcripts, capturing essential and relevant bits of student's responses to each question, was collected. The data were analyzed and result of the study can be described as follows; it was found that learners only think of the manipulative aspect when solving problems on limits and not of the limit concept, confusion over the concept of the limit and value of function, and ambiguity regarding the formal definition of the limit of function. The possible cause of the misconceptions can be attributed to instrumental learning and lack of the sound knowledge in algebra which is cornerstone to understand the limit concept.

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1. INTRODUCTION

The study of calculus is one of the most fundamental topics in mathematics due to its widespread application in multidisciplinary fields. It is an essential tool employed in the pursuit of biological sciences, physical sciences, engineering and economics [1-3], within it, the concept of limit is considered fundamental to calculus [4,5,6]. The concept of limit is also important to define the concept of derivative, integral, and convergence or divergence [5]. For instance, Instantaneous velocity is the limit of average velocities; the slope of a tangent line to a curve is the limit of the slope of secant line; the area of a circle is the limit of areas of the inscribed polygon as the number of sides increases infinitely (Brokate, Manchanda & Siddiqi, 2019, p.39).

It will be impossible to understand the concept of continuity, derivative, and definite integral without conceptualizing the essential components of limits.Additionally, Swinyard(2011) has claimed that a more succinct interpretation of the calculus involves a detailed understanding of key components of the formal description of the limit function.

Despite its importance, limits concepts have been historically difficult for introductory calculus student [7,5,8] (Gürbüz & Agsu, 2018;Muzangwa & Chifamba,2012; Tall,1992) as they cannot elucidate the role of limit in providing the algebraic definition of derivative and definite integral as limit of sum[1,9] (Hankioniemi, 2006). The epistemological obstacles that students encountered in teaching and learning of limits were, "Confusion over whether a limit is actually reached, dynamism of limits and static character of limits" [10]. Additionally, relating the limit concept to continuity and derivative were complex for the students [10].

The incomplete understanding of limit may attribute to the conventional teaching and learning approaches that focuses more on instrumental understanding than relational understanding[11] resulting in viewing the limit concept on its own without any connection to differential and integral calculus. Aréchiga Maravillas et al., [1] also asserted that students could mechanically calculate the limit but failed to manifest its underlying conceptual understanding. Recent investigation had demonstrated that students were more successful in learning the limit of function operationally rather than relationally, moreover they memorized the mechanical operation to be successful in the exam (Axtell, 2006). The instrumental understanding of calculus provides an imperfect concept definition of the calculus in the cognitive system that generates space between the instrumental understanding and the relational understanding, which are potential sources of the misconception in calculus [12, 13, 14].

Misconceptions are often defined as misunderstandings of concepts [15, 16). Kamid et al [17] defines misconception as conceptual framework of the learners in the context of powerful perceptions that varies from what should be in general according to empirical standards. Misconceptions arise when learners incorrectly visualized the knowledge he gets from his everyday encounters, structured, informal interactions, and distortions of instructions he receives before he generates his own theories either formally or informally (Zulnaid & Oktavira, 2018).

Some of the misconceptions identified in the limit of the functions: students assumed that a function has to be defined at a point to have limit at that point, a function undefined at a certain point does not have limit, the value of the function at specific point is equal to the limit of the function at that point (17,10). Jordaan [18] investigated the engineering students' misconception of the limit concept in the mathematics course, he asserted that value of the limit at a point is equal to the function value at that point. Additionally, they consider seeking boundaries and replacing them as essentially the same thing.

A study by Makonye [19] concluded that students missing the principles of pre-calculus including algebra and functions are the potential source of the misconceptions. It has been experimentally demonstrated that about 65% of students are unable to distinguish between the algebra and calculus principles [19]. The other potential calculus errors and misconception as seen in previous studies are errors related to mathematical terms such as tangent, limits, derivative, and functions [20]. Makonye [19] asserted that incomplete interpretation of the terminology led to an incomplete understanding of the definitions by students. The everyday vocabulary or words contained in the cognitive structure of students disagree with some of the mathematical terms such as,' limits,' simply, it means stop, but it had distinct connotations mathematically(Tall,1980). In the sense of sequence reaching limits, the phrases such as 'tends to', 'approaches', or 'gets' are used, they assume the series cannot equal the limits [5]. Thus, the language uses in the textbook used to describe the limit also contribute to the possible cause of the misconceptions in the limit of function.

Recalling the researcher's experience as a teacher, he is cognizant about students' difficulty in understanding the limit of the function and some of the potential misconceptions in the process of teaching and learning. The students were able to calculate limits on a number of problems at the end of the unit, but they found it very difficult to explain the idea of a limit in their own words. The researcher became curious and wanted to explore the concept of limits furthermore. He wanted to examine the causes of difficulty in learning the limit of function; whether the difficulty is due to the abstract nature of concepts, the presence of misconceptions, or due to any other reasons.

In brief, it is clear from the comprehensive evidence in literature that the misconceptions occur in mathematics in general, specifically in calculus in the international context, but minimal scholarly debate in the context of Bhutan, despite enough anecdotal evidence. Therefore, my study ascertain the possible misconceptions in limit of function. This study also serves as stepping stone for remedying the identified misconception with effective instructional strategies.

1.1 Research Questions

This study addresses the following questions;

1. What kind of misconceptions do students have in the limit of a function?
2. What are their difficulties to understand the limit concept?

1.2 The Significance of the Study

The important role that limits play in Calculus is acknowledged by most educators and educationists, but it is a fact that the inherent nature of limits is quite complex. Most students find it difficult to understand this idea. Therefore, the identification of the students' misconceptions will have value in the following ways:

- It might lead to a better understanding of the students' thought processes and the quality of learning that takes place.
- Knowledge of misconceptions can be employed in planning more effective teaching strategies and methods.
- It can also be used to present richer learning experiences to the students.
- The identification of misconceptions can create worthwhile opportunities to enhance learning.
- Minimizing misconceptions and providing a better understanding of the idea of a limit.

2. MATERIALS AND METHODS

This study was conducted in sequential approach. First in a survey design the students' pre conception was assessed using the concept test and analyzed qualitatively. Based on the findings of the concept test, a single group design was subjected to semi-structured interview to corroborate the finding of the possible misconceptions from the concept test.

The focus of the study was to determine the possible misconceptions in learning the limit of the function by grade XI students. In doing so, all the students studying mathematics in grade XI from Samtse Higher Secondary School were taken as study population. Purposive sampling was employed in selecting students as respondent of the study. The students who selected were enrolled in science stream for the academic year 2018-2019.

The study employed two different types of data collection instruments. The first is the calculus concept test (pre-test) based on the section dealing with limits in calculus from the prescribed textbooks, was offered to all the students to identify errors and misconceptions. The second was semi-structured interview to substantiate the findings from the concept test.

3. RESULTS

3.1 Analysis of the Data from Concept Test

The concept test administered to 25 students was to ascertain their conception about the limit of function in calculus. The four questions were designed to unearth students' possible misconception while learning the limit and their relational understanding about the concepts. Table 1 is the summary of their findings.

Based on the analysis made on the data gathered through the test in the specified area, the observed difficulties and error type of students in the study are summarized as follows:

- Twenty two of the respondents (88%) evaluated the limit of functions by simply substituting the value of the function but need to confirm whether limits is simply the mechanical substitution or not.
- Twenty five of the respondents (100%) indicated that the value of limit of function is same as the value of the function.
- Ten of the respondents (44%) had a conceptual or relational understanding of the limit of a function. The remaining fourteen respondents (56%) had partially confirmed the misconception in the limit of function which can further confirm by in-depth interview.

3.2 Analysis of the Data from the Interviews

To further corroborate the finding for the possible existence of misconceptions, students were interviewed with semi-structured. The interview was conducted with 5 purposefully selected students. All the 5 students were interviewed for 5 minutes.

Misconception 1: Students believe that limit concept is about the mechanical substitution. They think about the manipulative aspects and do not focus on the concept of limit.

The respondents S1 and S2 were not able to state the ε, δ definition correctly. The epsilon-delta definition of the limit function is utterly difficult for some students. Respondents S3, S4 & S5 think that they can only get a limit if they are able to substitute for x in the function. The respondents intuitively visualize the limit concept from the instrumental aspects not from the conceptual dimension.

The aforementioned responses with an exception of S2, indicated that the limits don't exist or it is undefined because when we evaluate, we will get $\frac{0}{0}$. Students were also confused about the intermediate form $\frac{0}{0}$ and undefined function. Students were confused over the existence of limit and being defined, in particular most of the students think that limit at a point is same as the value of the function at point. From their responses, it is apparent that limit is not defined but the function is defined. One student responded that $x^5 - 2^5$ must be simplified but do not know how to simplify. He had forgotten the factor theorem; remainder theorem and long division.

4. DISCUSSION

The summary of the identified misconceptions were;

a) Students talk of a limit not being defined at a point when it is the function that is not defined at the point.
b) Students think only about the manipulative aspects and do not focus on the concept of a limit.

Most students think that a function has to be defined at a point to have a limit at that point, a function undefined at a certain point does not have a limit. Students assume that 'limits' actually means replacing the value at which the limit is to be found with the expression .The concept image of the function and limit for students is governed by algebraic manipulations and routine algebraic manipulation isolated from the fundamental principles, this degree of comprehension is known as," action view of limit" [21]. Sebsibe et al. [22] also argued that the knowledge generated from the routine mechanical computation resulted in viewing the value of function at a point similar to the existence of function at that specific point. The earlier research indicates that the misconceptions exist in the space between instrumental and relational understanding. Thus, students should relationally comprehend the limit concept, instead of mechanically memorizing the rules and formulae.

Students think only about the manipulative aspects and do not focus on the underlying concept of the limit of function. The use of formulae make calculation easier, but it does not promote relational understanding, instead it enhances instrumental understanding [23]. He recommended that the growth of students' conceptual understanding should go hand in

Table 1. Breakdown of students' choice to the questions

Item	Correct		Incorrect		NR		Remarks
	N	%	N	%	N	%	
1	22	88	3	12	0	0	Evaluated the limits by substitution
2	0	0	25	100	0	0	The value of limit at point is same as the value of function that specific point
3	0	0	23	92	2	8	Limit is not being defined at a point when it is the function that is not defined at a point
4	10	44	14	56	1	0	The concept image of limit is dominated by mechanical substituion

Table 2. Data from the interviews

Interviewer	Code	Interviewee responses
Explain the definition in your own words $\lim_{x\to a} f(x) = l$ What about l?	S1	*We deal* $\partial > 0$ *such that* $x - a > 0, f(x) - l > \epsilon$...
	S2	*I can't recall it...*
	S3	*There is a number an in the* $x-axis$ *and that number as it approaches* x*, there is again another number in the* $y-axis$ *which is* l *it also approaches* $f(x)$
	S4	... $f(x)$ *cannot pass the point* l *when* x *is approaching that point a.*
		Because there is not x *in* l*, we cannot find the limit.*
	S5	*The* $lim_{x\to a} f(x)$ *is the same as* $f(a)$

Table 3. Interviewees' responses

Student	Interviewer	Interviewees' Responses
S1	Explain the method that you will use to find the limit ; $\lim_{x\to 2}\frac{x^5-2^5}{x-2}$	*"We have to substitute the value of x in the expression. When we substitute, we will get $\frac{0}{0}$, so limit is undefined and doesn't exist at $x=2$"*
	Is your function defined at $x=2$?	*"Function is defined at $x=2$ but limit is not defined"*
S2	Explain the method that you will use to find the limit ; $\lim_{x\to 2}\frac{x^5-2^5}{x-2}$	*"This one we can't immediately plug in 2. We have to find a way of factorizing the numerator so that it could cancel out with the $(x-2)$"*
	Can you get the limit if you can't factorize the numerator?	*"I can't find the limit, (The student is unable to factorize the numerator."*
S3	Explain the method that you will use to find the limit ; $\lim_{x\to 2}\frac{x^5-2^5}{x-2}$	*"We cannot find the value of limit because of its infinity. The limits don't exist"*
	Is your function defined at $x=2$?	*"The function is defined if we factorize the numerator"*
S4	Explain the method that you will use to find the limit ; $\lim_{x\to 2}\frac{x^5-2^5}{x-2}$	*"We cannot find the value of limit because the limit is undefined".*
	Is your function defined at $x=2$?	*" I forgot the concept of function"*

hand with the growth of their manipulative abilities. Recent research has shown that about 96 percent of participants are symbolically successful at overcoming the limit problems, their approach to solving the limit problem was dominated by procedural skills [24].

Some students attempted to simplify an expression $x^5 - 2^5$, before evaluating the limit, however, they were unsuccessful as they lacked pre-algebraic skills. Their failure to simplify an expression using the remainder and factor theorem became one of the potential sources of misconceptions. According to Siyepu [25], the poor simplifying skills of students shall result in possible errors and misconceptions.

From the interviewee's responses, many students apparently revealed having problems with understanding the definition of the limit and cannot state it correctly. Application of the definition of the limit is most problematic when students solve problems. The ε, δ definition to be considerably confusing to students. The formal definition of the limit is very difficult for students to state with understanding because of the epsilon (ε), delta (δ) approach. The student most probably did not understand the definition when it was taught. The student has learned the concept, but he could not put it in his own words. A few students have a concept image of the limit as it was evident when they attempted question number two correctly but lacked the knowledge of the conception definition of limits. English could be a problem in this instance and this hinders his grasp of the mathematical language and the ability to communicate. In research by Davies and Vinner (1986) it was reported that some of the usual misconceptions that students have resulted from the influence of language. Thus, one of the possible causes of misconception may be lack of understanding of mathematical language.

5. CONCLUSION

In conclusion, it is clear from the volume of evidence in literature that students' misconceptions concerning evaluation of limits i.e. limit is equal to the function value at a point (limit can be found by a method of substitution), students think of limit not being defined at a point when it is the function that is not defined at that point and definition of limits. The finding reveals that limits simply entail substituting the value at which the limit is to be found, into the expression.

CONSENT AND ETHICAL APPROVAL

As per international standard or university standard guideline participant consent and ethical approval has been collected and preserved by the authors.

COMPETING INTERESTS

Author has declared that no competing interests exist.

REFERENCES

1. Aréchiga Maravillas JC, Díaz SA, Salazar-Torres J, Andrade DA. Testing GeoGebra as an effective tool to improve the understanding of the concept of limit on engineering students. Journal of Physics: Conference Series. 2019;1408: 012008.
   Available:https://doi.org/10.1088/1742-6596/1408/1/012008
2. Mendezabal MJ, Tindowen DJ. Improving students' attitude, conceptual understanding and procedural skills in differential calculus through Microsoft mathematics. Journal of Technology and Science Education. 2018;8(4):385.
   Available:https://doi.org/10.3926/jotse.356
3. Sari P. GeoGebra as a means for understanding limit concepts. Southeast Asian Mathematics Education Journal. 2017;7(2):71-84.
   Available:https://doi.org/10.46517/seamej.v7i2.55
4. Agustyani AR, Putra AA, Nasution ML. Learners' difficulties and types of errors in topic of limit. Proceedings of the 3rd Asian Education Symposium (AES 2018); 2019.
   Available:https://doi.org/10.2991/aes-18.2019.77
5. Denbel DG. Students' misconceptions of the limit concept in a first calculus course. Journal of Education and Practice. 2014;5(34).
6. Hutkemri EZ. Undefined. Mediterranean Journal of Social Sciences; 2014
   Available:https://doi.org/10.5901/mjss.2014.v5n23p873
7. Cappetta RW, Zollman A. Agents of change in promoting reflective abstraction: a quasi-experimental, study on limits in college calculus. Journal of Research in

Mathematics Education. 2013;2(3):343-357.

Available:http://dx.doi.org/10.4471/redimat.2013.35

8. Liang S. Teaching the concept of limit by using conceptual conflict strategy and desmos graphing calculator. International Journal of Research in Education and Science. 2015;2(1):35.

   Available:https://doi.org/10.21890/ijres.62743

9. Orton A. Students' understanding of differentiation. Educational Studies in Mathematics. 1983;14(3):235-250.

   Available:https://doi.org/10.1007/bf00410540

10. Thabane, J. L. (1998). *Stduents' Understanding of the Limit Concept in a First year Calculus Course* [Unpublished master's thesis]. University of Witwatersrand.

11. Zachariades T, Pamfilos P, Christou O, Maleev R, Jones K. Teaching Introductory Calculus: approaching key ideas with dynamic software [Paper presentation]. CETL–MSOR Conference 2007 on Excellence in the Teaching & Learning of Maths, Stats & OR, University of Birmingham; 2007.

12. Hiebert J. Conceptual and procedural knowledge: The case of mathematics. Routledge; 2013.

13. Luneta K, Makonye PJ. Learners' Errors and misconceptions in elementary analysis: A case study of grade 12 class in South Africa. Acta Didatica Napocensia. 2010; 3(3):36-49.

14. Skemp RR. Relational understanding and instrumental understanding. Mathematics Teaching in the Middle School. 2006;12(2):88-95.

    Available:https://doi.org/10.5951/mtms.12.2.0088

15. Rittle-Johnson B, Schneider M, Star JR. Not a one-way street: Bidirectional relations between procedural and conceptual knowledge of mathematics. Educational Psychology Review. 2015;27(4):587-597.

    Available:https://doi.org/10.1007/s10648-015-9302-x

16. Van Dooren W, Lehtinen E, Verschaffel L. Unraveling the gap between natural and rational numbers. Learning and Instruction. 2015;37:1-4.

    Available:https://doi.org/10.1016/j.learninstruc.2015.01.001

17. Kamid K, Anwar K, Syaiful S, Sofnidar S, Liani L, Kurniawan W. Investigation into first-year college students' misconceptions about limit concept: A case study based on cognitive style. Universal Journal of Educational Research. 2020;8(4):1445-1452. Available:https://doi.org/10.13189/ujer.2020.080437

18. Jordaa T. Misconceptions of the limit concept in mathematics course for engineering students [Unpublished master's thesis]. University of South Africa; 2005.

19. Makonye P. Learners' mathematical errors in introductory differential calculus tasks: a study of misconceptions in senior secondary certificate examinations [Unpublished doctoral dissertation]. University of Johannesburg; 2011.

20. Makonye JP, Luneta K. Mathematical errors in differential calculus tasks in the senior school certificate examinations in South Africa. Education as Change. 2013;18(1):119-136. Available:https://doi.org/10.1080/16823206.2013.847014

21. Carlson M, Oehrtman M, Engelke N. The pre calculus concept assessment: A tool for assessing students' reasoning abilities and understandings. Cognition and Instruction. 2010;28(2):113-145.

    Available:https://doi.org/10.1080/07370001003676587

22. Sebsibe AS, Bekana T, Dorra BT. Effect of error analysis and concept change strategy on students understanding of calculus concepts. IEEE-SEM. 2019;7(7):369-385.

23. Bezuidenhout J. Limits and continuity: Some conceptions of first-year students. International Journal of Mathematical Education in Science and Technology. 2001;32(4):487-500. Available:https://doi.org/10.1080/00207390010022590

24. Sebsibe AS, Feza NN. Assessment of students' conceptual knowledge in limit of functions. International Electronic

Journal of Mathematics Education. 2019;15(2). Available:https://doi.org/10.29333/iejme/6294

25. Siyepu SW. Undefined. International Journal of STEM Education. 2015;2(1). Available:https://doi.org/10.1186/s40594-015-0029-5

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