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Refinement of Newton Gravitation Law

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Abstract

According to the theory of general relativity and experiments with atomic clocks in gravitation field, presence of the field shall cause time dilation of clock at rest in the field. This means that the gravitation constant G is not a true physical constant, but rather a function of the location of the setup in the field when measuring the parameter. This is because the definition of G includes a unit of time, and duration of that time unit is influenced by clock's location in the field. However, the theory assumes *a prior* that G shall remain constant in gravitation field, even though this may not be the case. On the other hand, relativistic gravitation phenomena can be derived without contradiction from a refined version of Newton's law of gravitation that complies with Einstein's law of mass-energy equivalence.

Keywords

General Relativity Theory, Gravitation, Planck Constant

1. Introduction

General relativity theory (GRT) [1] predicts various gravitational effects such as time dilation of clock in gravitation field [2] [3], redshift of photons [4] [5], deflection of photons [6], anomalous precession of moving objects [7], and existence of gravitation waves [8]. These predictions have all been observed and confirmed through experiments and/or observations [9]. However, the implications of time dilation caused by gravity have not been thoroughly explored beyond being considered as a "relativistic effect".

Metrologically, time dilation refers to the physical phenomenon that duration of unit of time becomes longer in comparison to that in reference state chosen for time comparison. Nowadays, dilation of time defined on atomic clock (atomic time, AT) is a well-established scientific fact beyond reasonable doubt, regardless of theory or interpretation. Therefore, on firm ground of experiment/observation, duration of unit of AT in static gravitation field (SGF) is not an invariant but function of the field experienced by the clock defining the AT. However, gravitation constant G is a function of duration of unit of AT by definition of G. Hence, there raises the question: would G still be constant, more precisely, state invariant (SIT), as was assumed/regarded? Further, many other physical units, constants, parameters are in direct/indirect association with unit of AT, e.g., speed of light in vacuo (SLV), Planck constant, etc. Then, would such entities be SIT as was/is assumed/regarded/understood while unit of AT is not? What might be implication/ramification if some physical units, constants, parameters are not SIT? Still further, plurality of laws of physics (LOP) involves *time*. Then, what might be impact/consequence if *time* is not SIT? These are some of the subjects of this analysis.

2. Atomic Clock in Static Gravitation Field

Consider a mass particle at rest in Rest Frame (RF) [10]. According to the law of Newton gravitation (LNG), exists SGF in association with the particle centered at location of the particle in RF. RF with SGF is referred to herein as *g*-frame, a rest frame by definition. From perspective of some other frame of reference, *g*-frame may not be identical in every aspect/detail to RF in absence of the field. That is, as viewed from some other reference frame, a spacial point of *g*-frame may not be coinciding with the corresponding one of RF in absence of field and/or path length between a pair of points of *g*-frame may not be identical to that of corresponding pair of points of RF in absence of field. However, any reference frame has to acknowledge that length of unit of length of RF must undergo exactly the same transformation, if any, to become length of unit of length of *g*-frame, whatever the transformation may be and regardless of dependency of such may have on location, orientation, etc. Therefore, any reference frame has to admit that *g*-frame as measured by unit of length of *g*-frame is and must be identical in every aspect/detail to the RF that *g*-frame is constructed from. Therefore, RF with SGF must be identical in every aspect/detail to RF in absence of the field if measured from within. Therefore,

$$L_{f,0,g} = L_{\boldsymbol{i}},\ \mathbb{U}_{L,f,0,g} = \mathbb{U}_{L,\boldsymbol{i}} \ \rightarrow L \ \&\ \mathbb{U}_L \subset \text{Field Invariant}\,. \quad (1)$$

$L_{f,0,g}$: Length of object at rest in field as measured at rest in same. L_i: Corresponding length of object at rest in RF as measured at rest in same. $\mathbb{U}_{L,f,0,g}$: Unit of length of *g*-frame at rest in same. $\mathbb{U}_{L,i}$: Unit of length of RF at rest in same.

That is, length and unit of length is field invariant by metrological test, *i.e.*, length aspect of object at rest in *g*-frame is one and same as that in RF regardless of presence/absence of field. Therefore, length aspect of an object is invariant to location of the object in field. In other words, presence of gravitation field shall not cause dilation/contraction of space nor object therein.

Consider a mass particle at rest in *g*-frame with center of the *g*-field assigned as origin. According to LNG, the particle shall experience gravitation force caused by the *g*-field. Under infinite space approximation (ISA),

$$\boldsymbol{F} = -\frac{GMm}{r^2}\hat{\boldsymbol{r}} \rightarrow \boldsymbol{F}_{p,x} = -\frac{G_{f,x}^{1/2} M_{f,x} G_{p,x}^{1/2} m_{p,x}}{r^2}\hat{\boldsymbol{r}},\ x = f,0,g\ . \tag{2}$$

$\boldsymbol{F}_{p,x}$: Gravitation force experienced by mass particle at rest in field. $G_{f,x}$: Gravitation constant G as perceived by field causing object at rest in field of others. $G_{p,x}$: G as perceived by mass particle at rest in field of others. $M_{f,x}$: Restmass of field causing object at rest in field of others as measured at location of same in same. $m_{p,x}$: Restmass of mass particle at rest in field of others as measured at location of same in same. r: Distance between mass particle and field center. $\hat{\boldsymbol{r}}$: Unit vector of particle location in field. x : State indicator.

Consider relocating the mass particle in field with infinitesimal velocity. Work done to relocate the particle is then

$$dw_{p,x} = -\boldsymbol{F}_{p,x} \cdot d\boldsymbol{r} \quad \rightarrow \quad \frac{dw_{p,x}}{dr} = \frac{G_{f,x}^{1/2} M_{f,x} G_{p,x}^{1/2} m_{p,x}}{r^2}\ . \tag{3}$$

$dw_{p,x}$: Infinitesimal work done to mass particle at rest in field with infinitesimal velocity and displacement.

Work done to the mass particle in such manner is gained by the particle as increment of restenergy of same. By the law of energy conservation (LEC),

$$dE_{p,x} = dw_{p,x} \quad \rightarrow \quad \frac{dE_{p,x}}{dr} = \frac{G_{f,x}^{1/2} M_{f,x} G_{p,x}^{1/2} m_{p,x}}{r^2}\ . \tag{4}$$

$E_{p,x}$: Restenergy of particle at rest in field at location of same in same.

By the law of mass-energy equivalence (LME) [11],

$$E = mc^2 \ \rightarrow \ E_{p,x} = m_{p,x} c_{p,x}^2 \ \rightarrow \ \frac{dE_{p,x}}{dr} = E_{p,x}\frac{G_{f,x}^{1/2} G_{p,x}^{1/2} M_{f,x}}{c_i^2 \beta_{p,g}^2 r^2},\ \beta_{p,g} \equiv \frac{c_{p,x}}{c_i}\ . \tag{5}$$

$c_{p,x}$: SLV as measured at location of mass particle at rest in field of others. c_i : SLV as measured/defined at rest in RF free of field of any kind.

Restenergy of particle and selfenergy of same at rest is identical/equivalent/indifferent regardless of presence/absence of field. Therefore,

$$E_{p,s,0,g} = E_{p,x} \rightarrow \frac{d\ln E_{p,s,0,g}}{dr} = \frac{G_{f,x}^{1/2} G_{p,x}^{1/2} M_{f,x}}{c_i^2 \beta_{p,g}^2 r^2} \rightarrow \frac{d\ln \Delta E_{p,s,0,g}}{dr} = \frac{G_{f,x}^{1/2} G_{p,x}^{1/2} M_{f,x}}{c_i^2 \beta_{p,g}^2 r^2} \tag{6}$$

$E_{p,s,0,g}$: Selfenergy of particle at rest in field. $\Delta E_{p,s,0,g}$: Selfenergy difference of particle at rest in field.

Suppose the particle in consideration is the one defining atomic clock at rest at location of the particle in field. By definition of AT [10] [12],

$$\mathbb{U}_{\mathrm{AT},x} = \frac{\mathcal{N}_t h_{p,x}}{\Delta E_{p,s,0,g}} \ \rightarrow \ \frac{d}{dr}\frac{h_{p,x}}{\mathbb{U}_{\mathrm{AT},x}} = \frac{h_{p,x}}{\mathbb{U}_{\mathrm{AT},x}}\frac{G_{f,x}^{1/2} G_{p,x}^{1/2} M_{f,x}}{c_i^2 \beta_{p,g}^2 r^2}\ . \tag{7}$$

$\mathbb{U}_{\mathrm{AT},x}$: Unit of AT of particle at rest in field. $\mathcal{N}_t$: Immutable numeral assigned by definition of unit of AT. $h_{p,x}$: Planck constant as measured at rest at location of particle in field.

By definition of SLV [13], with Expression (1),

$$\frac{1}{\mathbb{U}_{\mathrm{AT},x}} = \frac{c_{p,x}}{\mathcal{N}_c \mathbb{U}_{L,x}} \ \rightarrow \ \frac{1}{h_{p,x}}\frac{d}{dr}\left(h_{p,x}\beta_{p,g}\right) = \frac{G_{f,x}^{1/2} G_{p,x}^{1/2} M_{f,x}}{c_i^2 \beta_{p,g} r^2}\ . \tag{8}$$

$\mathcal{N}_c$: Immutable numeral assigned by definition of SLV.

Assumption 1: Static Planck constant is field invariant,

$$h_{p,x} = h_i \equiv h \subset \text{Field Invariant} . \tag{9}$$

h_i : Planck constant as measured at rest in RF free of field of any kind.

That is, it is **assumed** that outcome of measurement of static Planck constant shall not be affected by presence/absence of field regardless of strength of field, as long as field is static in RF and setup for measurement is at rest in field.

Under **Assumption 1**, Equation (8) becomes

$$\frac{d\beta_{p,g}}{dr} = \frac{G_{f,x}^{1/2} G_{p,x}^{1/2} M_{f,x}}{c_i^2 \beta_{p,g} r^2} . \tag{10}$$

From Equation (5),

$$\frac{dE_{p,x}}{d\beta_{p,g}} \frac{d\beta_{p,g}}{dr} = E_{p,x} \frac{G_{f,x}^{1/2} G_{p,x}^{1/2} M_{f,x}}{c_i^2 \beta_{p,g}^2 r^2} \to \frac{dE_{p,x}}{d\beta_{p,g}} = \frac{E_{p,x}}{\beta_{p,g}} \to E_{p,x} = \beta_{p,g} E_{p,i} . \tag{11}$$

$E_{p,i}$: Restenergy of mass particle at rest in RF free of field of others.

By LME,

$$E_{p,x} = m_{p,x} c_{p,x}^2 \to m_{p,x} = m_{p,i} / \beta_{p,g} . \tag{12}$$

$m_{p,i}$: Restmass of mass particle at rest in RF free of field of others.

By specification, the mass object causing SGF is in Rest State (RS) [10]. Therefore,

$$\begin{matrix} M_{f,x} = M_i \\ G_{f,x} = G_i \end{matrix} \to \frac{d\beta_{p,g}}{d\rho} = \frac{G_{p,x}^{1/2}}{G_i^{1/2}} \frac{1}{\beta_{p,g}\rho^2}, \ \rho \equiv \frac{r}{r_g}, \ r_g \equiv \frac{G_i M_i}{c_i^2} . \tag{13}$$

M_i : Restmass of field causing object at rest in RF free of field of others. G_i : Gravitation constant as measured at rest in RF free of any field. r_g: Characteristic length of the field (CLF). ρ: Distance between particle and field center, in unit of CLF.

Assumption 2: Static gravitation constant is state function of SGF in form

$$G_{p,x} = \beta_{p,g}^{2n_g} G_i . \tag{14}$$

n_g: Parameter of the state function, to be determined by experiment/observation.

This assumption suggests that static gravitation constant G may not be truly constant, but could vary depending on conditions of the measurement setup. It further proposes a specific form for this variability, but leaving the precise parameter to be determined through experiments and/or observations. Therefore, if it is confirmed through experimentation that $n_g = 0$, as certain gravity theories (including GRT) assume, then measurement of static gravitation constant G shall not be affected by location of the setup in gravitation field, as long as the field remains static in RF and the setup is stationary in the field. Accordingly, $G_{p,x}$ shall be identical/one and same/invariant whether measurement of G is conducted in far field (field strength approaching none) or vicinity of black hole, as long as field and measurement setup are both at rest with respect to RF. Conversely, if it is confirmed/verified by experiment/observation that $n_g \neq 0$ then outcome of measurement of $G_{p,x}$ shall be affected by where in field the setup for the measurement is located even if field is static in RF and setup for measurement is at rest in field.

Under **Assumption 2**, Equation (13) becomes

$$\frac{d\beta_{p,g}}{\beta_{p,g}^{n_g-1}}=\frac{d\rho}{\rho^2}\rightarrow\beta_g\equiv\beta_{p,g}=\begin{cases}\left(1-\left(2-n_g\right)/\rho\right)^{1/\left(2-n_g\right)}, & n_g\neq 2\\ e^{-1/\rho}, & n_g=2\end{cases}. \tag{15}$$

Therefore,

$$\rho_{\mathrm{SS}}=2-n_g\geq 0,\ \beta_g\Big|_{n_g=0}=\left(1-2/\rho\right)^{1/2},\ \beta_g\Big|_{n_g=-2}=\left(1-4/\rho\right)^{1/4}. \tag{16}$$

ρ_{SS}: Radius of Schwarzschild Sphere in unit of CLF.

In case n_g = 0, the expression for β_g is identical to the factor uncovered by Schwarzschild in exact solution of Einstein field equation for spherical SGF [14] [15]. Therefore, β_g is referred to hereinafter as Schwarzschild Factor. By definition in Expression (5), β_g is and must be real, nonnegative, and finite. In particular, collection of all locations where β_g = 0 is known as Schwarzschild Sphere (SS), event horizon [16], etc., and region of space enclosed by SS as Schwarzschild black hole (SBH) [17].

By definition of AT, under **Assumption 1** (conditioned AT, CAT), from Expression (11),

$$\frac{\mathbb{U}_{\mathrm{AT},x}}{\mathbb{U}_{\mathrm{AT},\boldsymbol{i}}}=\frac{\Delta E_{s,\mathrm{i}}}{\Delta E_{\mathrm{s},0,g}}=\frac{1}{\beta_g}\rightarrow\mathbb{U}_{\mathrm{AT},x}\Big|_{\rho_{\mathrm{SS}}\leq\rho<\infty}>\mathbb{U}_{\mathrm{AT},\boldsymbol{i}}. \tag{17}$$

$\mathbb{U}_{\mathrm{AT},i}$: Unit of Rest Time (RT) [10] defined on atomic clock in RS (RAT).

That is, time dilation shall occur to atomic clock at rest in SGF outside SS. In other words, duration of unit of selftime (ST) of particle defined on AT (SAT) at rest in field shall be longer than that of same at rest in RF free of any field except self-field (SF). Such phenomenon is commonly known as gravitation time dilation and predicted by GRT, wherein, **Assumption 1** and **Assumption 2** with n_g = 0 were assumed *a prior*, albeit implicitly.

According to Equation (17), duration of unit of SAT of mass particle at rest in field shall be approaching infinity towards SS and become so at SS. Therefore, if a mass particle is at rest in vicinity of SS then SAT of the particle shall advance slower than otherwise and cease to work at SS, *i.e.*, next clock event shall never occur hence the event is no longer recurring hence no longer clock event. However, lacking of a particular set of recurring events, by itself, shall have no effect to other events of the particle, *i.e.*, such by itself shall not prevent other events of the particle from happening, since events happen or not happen regardless of status of appointed set of recurring events [10].

Local AT in field, referred to as field AT, is realizable in *g*-frame, for example by placing one and each particle clock at one and each point in field. However, such AT is typically not able to be synchronized across various points in field. This is because ticking rates of the clocks, which are identical in absence of the field, vary at different locations of *g*-frame due to time dilation of the clocks caused by presence of the field, as explained by Equation (17). Therefore, in general, there is no such thing as locally defined common AT of *g*-frame. Consequently, nonlocal simultaneity of events in field on AT is, in general, indefinable hence the concept meaningless in metrology. Therefore, in general, field AT is not differentiable (in mathematical sense) with respect to location in field, and vice versa. For same reason, if SGF is regarded as a single entity then there is no such thing as SAT of the field. That is, there can have local AT in field but no common AT of field.

Locally defined common AT can exist in certain subset of *g*-frame, wherein, distance of spacial points of the subset to field center is identical hence ticking rates of otherwise identical atomic clocks located in such subset are identical hence synchronization among the clocks feasible, at least in principle. However, subsets having different distance to field center shall have no common AT among them even though each such subset may have its own common AT.

3. Speed of Light in Vacuo in Static Gravitation Field

SLV is defined on AT [18]. Therefore, from Expression (5),

$$c_{p,x} = \beta_g c_i \;\rightarrow\; c_{p,x}\Big|_{\rho_{SS} \le \rho < \infty} < c_i\,. \tag{18}$$

That is, SLV as measured by particle at rest in SGF outside SS shall be slower than that as measured by same in RS. In other words, SLV is not genuine physical constant in field but function of location of setup in field for measurement of SLV, regardless of **Assumption 2**. In contrast, GRT assumed *a prior* that SLV is universal constant regardless of presence/absence of field, which was originated from constancy of SLV as one of the founding principles of special relativity theory [19].

From Equation (18), SLV of mass particle at rest in field shall approach zero towards SS and become so at SS. Therefore, if a mass particle is at rest in vicinity of SS then SLV as measured by the particle on field AT shall be slower than that as measured away from SS. Therefore, if a mass particle is in motion in vicinity of SS then velocity of the particle shall be slower than that in region away from SS even if it is approaching local SLV in field. Further, if the particle were to land at SS then magnitude of touchdown velocity of the particle cannot exceed local SLV hence must be zero. Conversely, if a mass particle were to escape from SS then magnitude of its take-off velocity cannot exceed zero hence the particle cannot escape from SS. Such phenomenon is a consequence of Schwarzschild Factor being zero, regardless of status of SAT of the particle.

Likewise, if photon is approaching SS then velocity of photon shall be approaching zero towards SS and become so at SS regardless of definition of ST of photon. If photon arrives at SS then its landing velocity at SS must be zero. Conversely, if photon created at and/or from within SS were to leave SS then magnitude of its take-off velocity must be zero hence photon cannot escape from SS. Therefore, SS shall be observed as an optical black hole, *i.e.*, region of space whereat no photon of any kind can be emitted from nor reflected by.

Therefore, appearance of optical blackness of SS is not due to photon being attracted by gravity of SBH but caused by SLV being zero at SS. Gravity is force and photon is particle in motion at SLV. According to LME, there can be no interaction of any kind between any force and any particle in motion at SLV [10]. Alternatively, per Equation (2), static gravitation force is associated with restmass of object while photon does not and cannot have restmass by definition of photon, or equivalently, restmass of photon is/must be zero. Therefore, by LNG, there can be no interaction between photon and gravity.

On the other hand, according to Maxwell electrodynamics [20], without prejudice of any kind towards electrics or magnetics,

$$\varepsilon_0\mu_0 c^2 = 1 \rightarrow \frac{\varepsilon_{0,i}\mu_{0,i}c_i^2}{\varepsilon_{0,x}\mu_{0,x}c_x^2} = 1 \rightarrow \frac{\varepsilon_{0,i}}{\varepsilon_{0,x}} = \frac{\mu_{0,i}}{\mu_{0,x}} = \frac{c_x}{c_i} \equiv \beta_x . \tag{19}$$

ε_0: Electric constant, also known as vacuum permittivity. μ_0: Magnetic constant, also known as vacuum permeability. x: State indicator.

That is, SLV is but a function of property of vacuum. Therefore, according to Equation (19), presence of field in vacuum shall have physical effect to property of vacuum in field. It is under such context that gravity may be said as having interaction with photon, *i.e.*, presence of field in vacuo causes alteration of property of vacuo hence SLV in field hence photon travel in vacuo.

4. Gravitation Redshift

From Equation (11),

$$E_{s,0,g} = E_{p,x} = \beta_g E_i = \beta_g E_{s,i} \rightarrow E_{s,0,g}\Big|_{\rho_{SS}\le\rho<\infty} < E_{s,i} . \tag{20}$$

E_i: Restenergy of particle in RS. $E_{s,i}$: Selfenergy of particle in RS.

That is, restenergy hence selfenergy of mass particle at rest in field is proportional to that of the particle at rest in RS by Schwarzschild Factor of the particle. Therefore, if a mass particle is at rest in vicinity of SS then selfenergy of the particle hence selfenergy difference of same shall be nearing zero. Further, at SS, the particle can neither emit nor absorb photon of any kind even if such is available thereat. Therefore, if mass particle is in motion in vicinity of SS then emission/absorption of photon shall not be effective channel for the particle to lose/gain energy. Therefore, outside vicinity of SS is zone of optical silence.

According to Equation (20), selfenergy hence selfenergy difference of mass particle at rest in field is less than that of same at rest in RF free of field of others. Therefore, under **Assumption 1**, wavelength of photon emitted by mass particle at rest in field, as measured by observer at rest at location further away from field center, shall be longer than that of reference photon created by identical particle via identical process at observer location. This is known commonly as gravitation redshift of photon. Likewise, if observer is at rest at relatively closer location with respect to field center then wavelength of the photon received by observer shall be shorter than that of the reference photon created by identical particle via identical process at observer location. This is known as gravitation blueshift of photon.

Gravitation red/blueshift of photon was predicted by GRT and often interpreted as photon losing/gaining energy by being at different locations in field hence possessing different amount of potential energy. In other words, photon travel inward field would gain energy and outward field lose energy, due to gravitation potential of photon in field. However, as previously discussed [10], there is no interaction between the field and photons according to LME/LNG, meaning that there is no such thing as gravitation potential for photons in the field.

In GRT, it is the existence of mass that causes alteration of curvature of spacetime, which, in turn, causes alteration of state of energy of photon thereat. According to GRT, therefore, it is the energy of photon that is altered by being present in gravitation field. However, SGF is conservative field. Therefore, by LEC, total energy of particle, hence that of photon, in field must be conserved entity, *i.e.*, invariant to location of particle in field, or otherwise LEC violation inevitable.

5. Restmass in Field

From Equation (12),

$$m_{s,0,g} = m_{p,x} = \beta_g^{-1} m_{\boldsymbol{i}} = \beta_g^{-1} m_{s,\boldsymbol{i}} \;\;\rightarrow\;\; m_{s,0,g}\Big|_{\rho_{SS} \le \rho < \infty} > m_{s,\boldsymbol{i}} \,. \tag{21}$$

m_i : Restmass of particle in RS. $m_{s,i}$: Selfmass of particle at rest in RS. $m_{s,0,g}$: Selfmass of particle at rest in field.

That is, under LME, restmass of a particle in SGF is not field invariant but state variant, *i.e.*, pending on location of particle in field. In other words, restmass of a particle in field of others shall become heavier than otherwise, due to LME. Therefore, per LNG, gravitation force exerted by the mass particle to its surrounding shall become stronger than otherwise. Further, selfmass of a particle at rest in field shall approach infinity towards SS and become so at SS. Therefore, if a mass particle is at rest in vicinity of SS then selfmass of the particle as measured by itself at rest at location of the particle in field shall be heavier than that away from SS. Therefore, if a mass particle is sufficiently close to SS then gravitation force of the particle shall become comparable to that of SS no matter how light the particle was and how heavy the SBH may be. Consequently, if mass particle is sufficiently close to SS then static condition of the field shall no longer hold, even if only briefly. Therefore, mass particle shall not land at SS as if SS were stationary. Instead, mass particle and SS shall merge with each other by motion of both parties towards each other, regardless of lightness of particle/heaviness of SS. Accordingly, merge of mass particle and SS shall cause disturbance of otherwise static field of SS hence dissipation of energy associated with such disturbance.

According to Equation (21), restmass of a mass particle at rest in field is function of location of the particle in field. Therefore, total restmass of a system composed of plurality of mass particles dispersed in space shall be lighter than that of an identical system composed of identical amount of identical particles but less dispersed in space. In extreme case, two tiny mass particles having sufficiently close distance inbetween shall cause gravitation field of the pair as strong as massive star or SBH, due to Schwarzschild Factor approaching zero. Such was not anticipated by LNG nor GRT but insisted by LME.

6. State Invariant

As analyzed, duration of unit of AT under **Assumption 1**, hence SLV defined on AT, is function of state of the unit therefore not constant *per se*. State variables include state of motion, presence/absence of field, perspective, geometry of space, etc. On the other hand, many physical attributes are in direct/indirect association with unit of AT and/or SLV. Therefore, in general, any attribute should be regarded as function of state of the attribute unless/until proven/verified/supported to be otherwise by definition, LOP, measurement, or assumption. Attribute has unit. Therefore, in general, unit of attribute should be regarded as function of state of the unit unless/until proven/verified/supported to be otherwise by definition, LOP, experiment, or assumption. It is thus beneficial to sort out those attributes/units that are SIT, *i.e.*, independent of state of entities they are in association with.

By metrological analysis [10] and from Expression (1),

$$L_x = L,\ \mathbb{U}_{L,x} = \mathbb{U}_L \ \rightarrow\ L\ \&\ \mathbb{U}_L \subset \mathrm{SIT} \ \rightarrow\ \measuredangle \subset \mathrm{SIT}\,. \tag{22}$$

x : State indicator. $\measuredangle$: Attribute of angle.

That is, length and unit of length are SIT by metrological analysis. Attribute of angle is function of length alone. Therefore, angle is also SIT. The state indicator x is coded as the follows,

$$\begin{matrix} x \equiv \text{Perspective}, & \text{State of Motion}, & \text{Environment} \\ \boldsymbol{i}:\ \text{RF} & 0:\ \text{At rest} & 0:\ \text{Field Free} \\ f:\ \text{Field} & u:\ \text{In motion} & g:\ \text{Gravitation} \\ s:\ \text{Self} & c:\ \text{At SLV} & e:\ \text{Electric field} \end{matrix}\,. \tag{23}$$

In particular, denote $\boldsymbol{i}$-state $\equiv$ $\boldsymbol{i}$, 0, 0 $\equiv$ $\boldsymbol{i}$. That is, state of being at rest in RF free of field of any kind except SF is referred to as $\boldsymbol{i}$-state or RS.

By **Assumption 1**,

$$h_x = h \ \rightarrow\ h \subset \mathrm{SIT}\,. \tag{24}$$

That is, Planck constant h is SIT by assumption.

By definition of unit of AT, with the law of Planck on photon energy (LPE), under **Assumption 1**,

$$\mathbb{U}_{\mathrm{AT},x} = \mathcal{N}_t h_x / \Delta E_{s,x} \ \rightarrow\ \Delta E_{s,x}\mathbb{U}_{\mathrm{AT},x} = \mathcal{N}_t h \ \rightarrow\ \Delta E_s \mathbb{U}_{\mathrm{AT}} \subset \mathrm{SIT}\,. \tag{25}$$

$\mathcal{N}_t$: Immutable numeral assigned by definition of unit of AT. ΔE_s : Selfenergy difference of particle defining unit of AT.

That is, composite attribute $\Delta E_s \mathbb{U}_{\mathrm{AT}}$ is SIT under CAT. ΔE_s is state variant. Therefore, $\mathbb{U}_{\mathrm{AT}}$ must be state variant. Unit of AT is defined on selfenergy of particle defining the time. Therefore, time thus defined is SAT of particle, *i.e.*, local AT of particle at rest at particle location in reference frame comoving with particle.

By definition of SLV, with Expression (22),

$$c_x \mathbb{U}_{t,x} = \mathcal{N}_c \mathbb{U}_L \;\rightarrow\; c\mathbb{U}_t \subset \text{SIT}\,. \tag{26}$$

That is, composite attribute $c\mathbb{U}_t$ is SIT by definition and analysis. Therefore, if SLV is defined on AT then, since $\mathbb{U}_{\text{AT}}$ is state variant under CAT, c must be state variant under same.

By definition, with the rule of numeration [10],

$$u_x \equiv \frac{v_x}{c_x} \equiv \frac{1}{c_x}\frac{ds_x}{dt_x} = \frac{ds}{c_x dt_x} = \frac{1}{c_i}\frac{ds}{dt_i} = \frac{1}{\mathcal{N}_c}\frac{dn_L}{dn_t} \;\rightarrow\; u \subset \text{SIT}\,. \tag{27}$$

That is, reduced velocity u is SIT.

For convenience, define some reduced attributes as follows,

$$\epsilon_x \equiv E_x / E_i,\; \mu_x \equiv m_x / m_i,\; \beta_x \equiv c_x / c_i\,. \tag{28}$$

ϵ, μ, β : Reduced energy, mass, SLV.

In reduced form, LME is expressed as

$$\epsilon_x = \beta_x^2 \mu_x\,. \tag{29}$$

As derived previously [10], for particle in motion in RF, from perspective of RF,

$$\epsilon_{i,u,0} = \beta_u^{-1},\; \mu_{i,u,0} = \beta_u^{-1} \;\rightarrow\; \beta_{i,u,0} = 1 \;\rightarrow\; \mathbb{U}_{t,i,u,0} / \mathbb{U}_{t,i} = 1\,. \tag{30}$$

β_u : Lorentz Factor, $\beta_u \equiv \sqrt{1-u^2}$.

For particle in motion in RF, from self-perspective of particle, *i.e.*, viewed from location of particle at rest in reference frame comoving with particle, under CAT,

$$\epsilon_{s,u,0} = \beta_u^{+1},\; \mu_{s,u,0} = \beta_u^{-1} \;\rightarrow\; \beta_{s,u,0} = \beta_u,\; \mathbb{U}_{\text{AT},s,u,0} / \mathbb{U}_{\text{AT},i} = \beta_u^{-1}\,. \tag{31}$$

For particle at rest in field, from perspective of field, *i.e.*, at rest at location of particle in *g*-frame, from Equation (20) and (21),

$$\epsilon_{f,0,g} = \beta_g^{+1},\; \mu_{f,0,g} = \beta_g^{-1} \;\rightarrow\; \beta_{f,0,g} = \beta_g,\; \mathbb{U}_{\text{AT},f,0,g} / \mathbb{U}_{\text{AT},i} = \beta_g^{-1}\,. \tag{32}$$

For particle at rest in field, self-perspective of particle is identical to that at rest at location of particle in *g*-frame. Therefore,

$$\epsilon_{s,0,g} = \beta_g^{+1},\; \mu_{s,0,g} = \beta_g^{-1} \;\rightarrow\; \beta_{s,0,g} = \beta_g,\; \mathbb{U}_{\text{AT},s,0,g} / \mathbb{U}_{\text{AT},i} = \beta_g^{-1}\,. \tag{33}$$

For motion of particle in field, effect of field is or is equivalent to physical constraint of non-rigid type. If particle is in circular motion with respect to field center then field acts as physical constraint counterbalancing perceptible centrifugal force caused by motion of particle. If particle were to move along radius direction then such motion is allowable by field due to non-rigidity of field confinement. If particle ceases motion then particle is at rest in field. Therefore, for particle in motion in field, from perspective of field, *i.e.*, perspective at rest in field at particle location, from Equations (30) and (32), under CAT,

$$\begin{aligned}&\begin{matrix}E_{f,u,g}=\beta_u^{-1}E_{f,0,g} \\ m_{f,u,g}=\beta_u^{-1}m_{f,0,g}\end{matrix},\ \begin{matrix}E_{f,0,g}=\beta_g^{+1}E_{\boldsymbol{i}} \\ m_{f,0,g}=\beta_g^{-1}m_{\boldsymbol{i}}\end{matrix} \rightarrow \\ &\begin{matrix}\epsilon_{f,u,g}=\beta_u^{-1}\beta_g^{+1} \\ \mu_{f,u,g}=\beta_u^{-1}\beta_g^{-1}\end{matrix} \rightarrow \beta_{f,u,g}=\beta_g \rightarrow \frac{\mathbb{U}_{\mathrm{AT},f,u,g}}{\mathbb{U}_{\mathrm{AT},\boldsymbol{i}}}=\frac{1}{\beta_g}\end{aligned}. \tag{34}$$

In self-perspective of particle in motion in field, from Equations (31) and (33), under CAT,

$$\begin{aligned}&\begin{matrix}E_{s,u,g}=\beta_u^{+1}E_{s,0,g} \\ m_{s,u,g}=\beta_u^{-1}m_{s,0,g}\end{matrix},\ \begin{matrix}E_{s,0,g}=\beta_g^{+1}E_{\boldsymbol{i}} \\ m_{s,0,g}=\beta_g^{-1}m_{\boldsymbol{i}}\end{matrix} \rightarrow \\ &\begin{matrix}\epsilon_{s,u,g}=\beta_u^{+1}\beta_g^{+1} \\ \mu_{s,u,g}=\beta_u^{-1}\beta_g^{-1}\end{matrix} \rightarrow \beta_{s,u,g}=\beta_u\beta_g \rightarrow \frac{\mathbb{U}_{\mathrm{AT},s,u,g}}{\mathbb{U}_{\mathrm{AT},\boldsymbol{i}}}=\frac{1}{\beta_u\beta_g}\end{aligned}. \tag{35}$$

In particular,

$$\epsilon_f \equiv \epsilon_{f,u,g}=\beta_g/\beta_u \subset \text{Field Invariant}. \tag{36}$$

That is, total energy of particle in motion in field as measured at rest at particle location in field is invariant to location of particle in field, ST of particle, state of motion of particle, geometry of space, etc., due to conservation nature of centripetal interaction between particle and field. Were this not the case, machine of perpetual motion could then be constructed. Therefore, total energy of particle in motion in field is field invariant by LOP.

For particle in motion at low speed in far field, Equation (36) can be approximated as, under ISA,

$$\begin{aligned}&\begin{matrix}u \ll 1 \\ \rho \gg \rho_{\mathrm{SS}}\end{matrix} \rightarrow \epsilon_f \simeq 1+\frac{u^2}{2}-\frac{1}{\rho} \rightarrow \\ &E_{f,u,g} \simeq m_{\boldsymbol{i}}c_{\boldsymbol{i}}^2+\frac{m_{\boldsymbol{i}}v_{\boldsymbol{i},u,0}^2}{2}-\frac{G_{\boldsymbol{i}}M_{\boldsymbol{i}}m_{\boldsymbol{i}}}{r}=mc^2+\frac{1}{2}mv^2-\frac{GMm}{r}\end{aligned}. \tag{37}$$

This is recognized as expression for total energy of particle in field under Newtonian approximation, in which, the first term is known as restenergy, second kinetic energy, and third potential energy of particle in field.

In general, state function of attribute is homogeneous, *i.e.*, being function of state variables but not that of the attribute,

$$a_x=f_a[x,y]a_y \rightarrow f_a[x,x]=1. \tag{38}$$

a: Attribute. x,y: State variables. f_a: State function of category of attribute a, pure numeral function of state variables alone.

Therefore, if

$$\begin{matrix}p\in\{a\} \\ q\in\{a\}\end{matrix} \rightarrow \begin{matrix}p_x=f_a[x,y]p_y \\ q_x=f_a[x,y]q_y\end{matrix} \rightarrow \frac{p_x}{q_x}=\frac{p_y}{q_y}=\frac{p}{q}\subset \mathrm{SIT},\ p,q\in\{a\}. \tag{39}$$

p,q: Attributes. $\{a\}$: Category of attribute a.

That is, if attributes belong to same category and are in same state then ratio of the attributes is SIT. This is referred to as rule of metrology (ROM). By definition of attribute [10],

$$a_x \equiv n_{a,x}\mathbb{U}_{a,x} \rightarrow n_{a,x} = a_x / \mathbb{U}_{a,x} \subset \text{SIT} \rightarrow a_x = n_a \mathbb{U}_{a,x}. \tag{40}$$

n_a : Numeral aspect of attribute a. $\mathbb{U}_a$: Unit of attribute a.

That is, numeral aspect of attribute is SIT if attribute is expressed in unit of attribute in same state as that of the attribute.

By Equation (38),

$$\begin{array}{l} E_{s,x} = f_E[x,\boldsymbol{i}]E_{s,\boldsymbol{i}} \\ \Delta E_{s,x} = f_E[x,\boldsymbol{i}]\Delta E_{s,\boldsymbol{i}} \end{array} \rightarrow \frac{E_{s,x}}{\Delta E_{s,x}} = \frac{E_{s,\boldsymbol{i}}}{\Delta E_{s,\boldsymbol{i}}} \rightarrow \frac{E_s}{\Delta E_s} \subset \text{SIT}. \tag{41}$$

E_s : Selfenergy of particle. ΔE_s : Selfenergy difference of particle.

Unit of energy is also energy. Therefore, by ROM,

$$\mathbb{U}_{E,x} = f_E[x,\boldsymbol{i}]\mathbb{U}_{E,\boldsymbol{i}} \rightarrow \frac{E_{s,x}}{\mathbb{U}_{E,x}} = \frac{E_{s,\boldsymbol{i}}}{\mathbb{U}_{E,\boldsymbol{i}}} \rightarrow \frac{E_s}{\mathbb{U}_E} \subset \text{SIT} \rightarrow \frac{\mathbb{U}_E}{\Delta E_s} \subset \text{SIT}. \tag{42}$$

$\mathbb{U}_E$: Unit of energy.

With Expressions (25) and (26), under CAT,

$$E_s \mathbb{U}_{\text{AT},s} \,\&\, \mathbb{U}_E \mathbb{U}_{\text{AT},s} \subset \text{SIT} \rightarrow E_s / c_s \subset \text{SIT} \rightarrow \epsilon_s = \beta_s^{+1},\ \mu_s = \beta_s^{-1}. \tag{43}$$

7. Michelson-Morley Experiment

Michelson-Morley experiment (MME) [21] can be described in language of particle as the follows:

1) Two identical photons are created by single event at single spot (denoted as origin) on Earth surface;

2) One of the photons is directed towards traveling horizontally to some distance L_1 in direction parallel to that of motion of Earth surface, reflected there instantly, and then returned to origin. Denote duration from creation of the photon at origin to arrival of the reflected photon at origin according to local time at origin as $\delta t_{1,/\!/} = L_1 c_{/\!/,+}^{-1} + L_1 c_{/\!/,-}^{-1}$;

3) The other photon is directed to travel horizontally to some distance L_2 in direction perpendicular to that of motion of Earth surface, reflected there instantly, and then returned to origin. Denote duration from creation of the photon at origin to arrival of the reflected photon at origin according to local time at origin as $\delta t_{1,\perp} = L_2 c_{\perp,+}^{-1} + L_2 c_{\perp,-}^{-1}$;

4) Denote the difference between the above durations as $\Delta t_1 \equiv \delta t_{1,/\!/} - \delta t_{1,\perp}$;

5) Swap the two distances above and repeat the measurement to obtain $\Delta t_2 \equiv \delta t_{2,\perp} - \delta t_{2,/\!/}$.

Then,

$$\Delta t_1 - \Delta t_2 = (L_1 + L_2)\left(c_{/\!/,+}^{-1} + c_{/\!/,-}^{-1} - c_{\perp,+}^{-1} - c_{\perp,-}^{-1}\right). \tag{44}$$

Therefore,

$$\textbf{If} \quad c_x = c_{\text{Origin}},\, x \in \{/\!/, \perp, \pm\} \quad \textbf{Then} \quad \Delta t_1 - \Delta t_2 = 0. \tag{45}$$

That is, if SLV is identical whether photon path is parallel, anti-parallel, or perpendicular to direction of motion of Earth surface then $\Delta t_1 = \Delta t_2$, *i.e.*, time delay at origin between the two photons returning from their respective paths is invariant to orientation of the test setup with respect to motion of Earth surface. Therefore, interference pattern of the two photons at origin is invariant to same, *i.e.*, null result of MME; otherwise $\Delta t_1 \neq \Delta t_2$.

Null result of MME is derivable from LME. For such purpose, approximate the photon path parallel and anti-parallel to motion of Earth surface as a section of geodesic (ignoring effects of gravity of Earth/Sun/Moon). Perform mass-energy conversion experiment of matter at rest at a point of the section and denote

$$\alpha \equiv \Delta E / \Delta m . \tag{46}$$

ΔE: Alteration of energy during single mass-energy conversion process of matter at rest at point in space. Δm: Alteration of mass during same mass-energy conversion process of matter at rest at same point in space.

Then, the measured attribute α is guaranteed by continuity, homogeneity, and isotropy specification of space to be identical, one and same, invariant, regardless of where at the section the measurement is conducted if the section is at rest with respect to RF. If the section is in uniform motion along its geodesic then α is still identical/one and same/invariant for the measurement anywhere at the section, although α thus measured may be different from that as measured at rest with respect to RF.

Expand the section along its perpendicular direction to become a two dimensional surface containing photon path perpendicular to motion of Earth surface (approximately). Thus, if the surface is at rest with respect to RF then measurement of α anywhere at the surface is identical/one and same/invariant. If the surface is in uniform motion along the geodesic and perpendicular extent of the surface is negligibly small comparing to radius of physical space then, up to precision of the approximation, measurement of α anywhere at the surface is still identical/one and same/invariant, although α thus measured may differ from that as measured at rest with respect to RF. According to LME,

$$c^2 = \alpha \ \rightarrow \ c = \sqrt{\alpha} . \tag{47}$$

That is, up to precision of the approximations aforementioned, SLV is identical anywhere at the two dimensional surface whether it is at rest or in uniform motion with respect to RF, although SLV determined under different state of motion of the surface may be different. Therefore, by LME, MME conducted at the surface shall have null result, to precision of the approximations, regardless of paths/directions of photons at the surface in the experiment.

There can or may have been vertical version of MME that shall also yield null result. In such experiment, one of the photon paths along horizontal directions of Earth surface in original MME is replaced by a path along vertical direction at test spot. By definition of velocity, with ROM and Expression (22),

$$\begin{gathered} \frac{c_x}{c_{\mathtt{i}}} = \frac{ds_x}{dt_x}\frac{dt_i}{ds_i} = \frac{ds_x}{\mathbb{U}_{L,x}}\frac{\mathbb{U}_{L,\mathtt{i}}}{ds_{\mathtt{i}}}\frac{\mathbb{U}_{L,x}}{\mathbb{U}_{L,i}}\frac{dt_i}{dt_x} = \frac{dt_i}{dt_x} \ \rightarrow \\ c_x dt_x = c_i dt_i = ds \ \rightarrow \ dt_i = \frac{ds}{c_i} \ \rightarrow \ \delta t_i = \frac{L}{c_i} \end{gathered} . \tag{48}$$

ds: Line element of trajectory of photon in vacuo. *L*: Length of photon path.

That is, time taken for photon to travel through path in vacuo is invariant to state of path as well as that of photon. Therefore, at origin of setup for MME, time delay between two photons sent out local simultaneously from origin and returned back to same via different paths is invariant to orientation of test setup with respect to Earth and state of motion thereof but only to path length. Therefore, interference pattern of such photons is invariant to same, *i.e.*, null result of MME, even though SLV along vertical path is function of elevation on Earth therefore different from that along horizontal path. Equation (48) is a general equation applicable to any state of motion/field environment. Therefore, any version of MME shall produce null result, without approximation.

8. Photon Energy in Field

Consider a pair of signal source and receiver at rest in field at elevation a and b, $a \neq b$, $\rho_{SS} < a$, $\rho_{SS} < b$, measured from field center in unit of CLF. Suppose source sends a set of signals via identical route towards receiver (event *A*) in duration $T_{A,a}$ according to local clock at a and receiver receives the same set of signals (event *B*) in duration $T_{B,b}$ according to local clock at *b*. From perspective of RF, duration of event *A* is $T_{A,\boldsymbol{i}}$ and that of event *B* $T_{B,\boldsymbol{i}}$, all according to RT. Then,

$$T_A \equiv T_{A,a} = T_{A,\boldsymbol{i}}, \quad T_B \equiv T_{B,b} = T_{B,\boldsymbol{i}}. \tag{49}$$

T_A: Duration of event *A*. $T_{A,a}$: T_A as measured by clock at source location. $T_{A,i}$: T_A as measured in RT. T_B: Duration of event *B*. $T_{B,b}$: T_B as measured by clock at receiver location. $T_{B,i}$: T_B as measured in RT.

As observed from perspective of RF, identical set of identical signals travels through identical path from identical point a to arrive at identical point *b*. Therefore,

$$T_{A,\boldsymbol{i}} = T_{B,\boldsymbol{i}} \rightarrow T_A = T_B. \tag{50}$$

By definition of *frequency*,

$$\nu_{A,x} \equiv (n-1)/T_{A,x}, \ \nu_{B,x} \equiv (n-1)/T_{B,x}, \ x \in \{a, b, \boldsymbol{i}\} \rightarrow \nu_{A,x} = \nu_{B,x}. \tag{51}$$

$\nu_{A,x}$: Frequency of event *A* as measured by clock at x. $\nu_{B,x}$: Frequency of event *B* as measured by clock at x. *n*: Number of signals (*the* event) sent by source and received by receiver.

That is, frequency of the event is invariant with respect to location of measurement in field if frequency is expressed in identical unit. Therefore, by LPE, under **Assumption 1**,

$$h_A \nu_{A,x} = (h_A / h_B) h_B \nu_{B,x} = h_B \nu_{B,x} \rightarrow E_{p,a,\boldsymbol{i}} = E_{p,b,\boldsymbol{i}} \rightarrow E_{p,f,0,g} = E_{p,\boldsymbol{i}}. \tag{52}$$

h: Planck constant, field invariant by **Assumption 1**. $E_{p,f,0,g}$: Energy of photon in field, measured as selfenergy increment of particle at rest in field upon absorption of the photon. $E_{p,j,i}$: Energy of photon in field at elevation *j* in terms of unit of energy in RS. $E_{p,i}$: Energy of photon in RF, measured as selfenergy increment of particle at rest in RF upon absorption of the photon. That is, energy of photon in field is invariant to location of same in same under **Assumption 1**. In other words, SGF shall not alter energy of photon traveling therein if **Assumption 1** is true.

On the other hand, by conservation nature of centripetal force field, total energy of particle, hence that of photon, in motion in field must be invariant in compliance with LEC. Alternatively, by LME, no interaction can exist between field and particle in motion at SLV [10]. Therefore, by LEC, energy of photon in field must be invariant. From Equation (51), frequency of photon in field is invariant to location of same in same. Therefore, by LPE, Planck constant must be field invariant. That is, **Assumption 1** is true with respect to centripetal force field.

9. Test of Time Dilation of Atomic Clock in SGF

Phenomenon of time dilation in SGF was predicted by GRT and Schwarzschild Factor obtained from GRT is identical to that for atomic clock in SGF in the case of $n_g = 0$ under ISA,

$$\beta_{\text{GRT}} \equiv \sqrt{1-\frac{2GM}{c^2 r}} = \sqrt{1-\frac{2G_i M_i}{c_i^2 r}} \equiv \sqrt{1-\frac{2}{\rho}} = \beta_g\big|_{n_g=0}. \tag{53}$$

Therefore, any test/experiment/measurement in the category of verifying GRT prediction on time dilation of atomic clock in SGF is equally effective in falsifying result of this analysis, understood that interpretation is different with respect to that of GRT. Such test may also facilitate in determination of parameter n_g in **Assumption 2**.

Among the tests of Expression (53), a well-known example may be Pound-Rebka experiment (PRE) [22]. In such, photon source is placed at relatively higher altitude and matching absorber at relatively lower altitude but same latitude and longitude on Earth and photon detector located behind absorber. Reversal of the configuration is equivalent in effect, *i.e.*, source can be placed at relatively lower altitude and absorber/detector at relatively higher altitude. If source and absorber are at same altitude then less photon shall penetrate through absorber, due to resonance absorption of the photons by matching absorber. If source and absorber are at different altitude in field then their respective energy gaps (difference in selfenergies of particles in states involved in quantum transition) shall become different due to difference in β_g associated with altitude of entity at rest at corresponding altitude. Thus, if absorber is at relatively lower altitude then photon emitted by source (at relatively higher altitude) shall be more energetic than corresponding energy gap of absorber; if source is at relatively lower altitude then photon emitted by source shall be less energetic than corresponding energy gap of absorber (at relatively higher altitude). In quantitative measurement, mechanical drive is installed either with source or with absorber, causing relative translation motion between source and absorber. If Doppler Effect caused by the mechanical modulation compensates for field caused energy mismatch then photon signal at detector is minimal.

Consider the configuration of photon source at relatively higher altitude with velocity modulation. Label such location as a and the corresponding lower altitude location as *b*. Approximate field of Earth at test site as SGF having spherical symmetry with respect to field center, then, according to Equation (35), under ISA,

$$\Delta E_{s,j} = \beta_{u,j}\beta_{g,j}\Delta E_{s,i},\ \beta_{u,j} \equiv \sqrt{1-u_j^2},\ \beta_{g,j} = \left(1-\frac{2-n_g}{\rho_j}\right)^{1/(2-n_g)},\ j \in \{a,b\}. \quad (54)$$

$\Delta E_{s,j}$: Selfenergy gap of particle in motion at altitude *j* with respect to field. u_j: Reduced velocity of motion of particle at altitude *j* with respect to field. $\Delta E_{s,i}$: Selfenergy gap of particle in RS. ρ_j: Distance between particle at altitude *j* and field center, in reduced unit.

From Equation (52), energy of photon in field is invariant if both source and receiver are at rest in field. Therefore, resonance condition of the test setup is (with approximations partially outlined)

$$E_{p,b} = \frac{E_{p,a}}{1+u_M} = \Delta E_{s,b} \rightarrow \frac{\beta_{u,a}\beta_{g,a}}{1+u_M} = \beta_{u,b}\beta_{g,b} \rightarrow \sqrt{1-u_b^2}\,\beta_{g,b} = \frac{\sqrt{1-u_a^2}}{1+u_M}\beta_{g,a}. \quad (55)$$

$E_{p,b}$: Energy of photon as absorbed by particle at rest at *b*. $E_{p,a}$: Energy of photon as emitted by particle in motion at a. u_M: Reduced inline resonance modulation velocity of particle at a, parting from b.

Under static approximation of Earth field, with parameters from Table 1,

$$u_a = u_M,\ u_b = 0 \rightarrow 2u_M = 2\left(\beta_{g,a}^2 - \beta_{g,b}^2\right)/\left(\beta_{g,a}^2 + \beta_{g,b}^2\right) \approx 4.924\times10^{-15}. \quad (56)$$

Test result as reported [23] was (0.997 ± 0.008) × 4.905 × 10^{-15}, with unspecified accuracy on the working height in the test setup. On the other hand, resolution of the test was insufficient in differentiation of n_g.

If effect of spin of Earth is factored in the PRE then

$$\boldsymbol{u}_a = \boldsymbol{u}_{a,\omega} + \boldsymbol{u}_M,\ \boldsymbol{u}_b = \boldsymbol{u}_{b,\omega},\ u_{j,\omega} \equiv r_j\omega_f\cos\theta/c_f. \quad (57)$$

$u_{j,\omega}$: Reduced velocity of particle in motion at altitude *j* due to spin of Earth. r_j: Distance between particle at altitude *j* and field center. ω_f: Angular velocity of spin of Earth with respect to Earth field. θ: Latitude of location of particle, with that of Equator defined as $\theta_E \equiv 0$. c_f: SLV as measured/defined on AT in field at particle location.

Modulation velocity of particle (inline motion) at a is orthogonal to spin velocity of Earth. Therefore,

$$u_a^2 = u_{a,\omega}^2 + 2\left(\boldsymbol{u}_{a,\omega}\cdot\boldsymbol{u}_M\right) + u_M^2 \rightarrow \frac{1-u_{a,\omega}^2-u_M^2}{\left(1+u_M\right)^2} = \left(1-u_{b,\omega}^2\right)\frac{\beta_{g,b}^2}{\beta_{g,a}^2} \equiv \lambda. \quad (58)$$

With parameters from Table 1,

$$2u_M = \left(\sqrt{1-(1+\lambda)u_{a,\omega}^2} - \lambda\right)/(1+\lambda) \approx 4.915\times10^{-15}. \quad (59)$$

Table 1. Parameters for PRE at Jefferson Physics Laboratory in Harvard University.

Item	Parameter
Latitude of a and b	42˚22′29″N[a]
Altitude of b	8 m[a, b]
Altitude of a	Altitude of b + 22.5 m[c]
GM_E	$3.986\ 004\ 415 \times 10^{14}$ m^3 s^{-2}[d]
Equatorial Radius of Earth	6 378 136.6 m[d]
Polar Radius of Earth	6 356 752.314 2m[f]
Sea Level at a and b	6 368 432 m[e]
ω_f	$7.292\ 115 \times 10^{-5}$ rad s^{-1}[d]

[a]Retrieved from Google Earth with aerial survey image dated April 23, 2018. [b]Altitude of b was assumed to have been at ground level of Tower of Jefferson Physics Laboratory in Harvard University. [c][23]. [d][24], TT-compatible. [e]Spheroid model of Earth was assumed to estimate sea level at the test site. [f][25].

That is, when effect of spin of Earth is considered, difference in measured and expected values of Earth field effect is further reduced. Nevertheless, the test result is still too coarse for n_g differentiation.

Notwithstanding above, PRE demonstrated that presence of field does cause alteration of selfenergy of particle at rest in field hence selfenergy difference of same in same. Therefore, time dilation of unit of SAT of particle at rest in field shall occur, since duration of unit of AT is function of selfenergy difference of particle defining AT.

10. Particle Motion in Static Gravitation Field

The original LNG as presented by Newton [26] can be expressed as

$$\frac{d\boldsymbol{P}_j}{dt} = -Gm_k m_j \boldsymbol{f}_{j,k}, \quad \boldsymbol{f}_{j,k} \equiv \left(\boldsymbol{r}_j - \boldsymbol{r}_k\right) / \left\|\boldsymbol{r}_j - \boldsymbol{r}_k\right\|^3. \tag{60}$$

$\boldsymbol{P}_j$: Momentum of particle j. t: Time. G: Gravitation constant. m: Mass of particle. $\boldsymbol{r}$: Location vector of particle.

Wherein, states of entities were unspecified and space was assumed Euclid. From context of Newton's original work, it can be inferred that the attributes involved in Equation (60) were viewed from perspective of an inertial reference frame at rest, hence equivalent to viewpoint of g-frame, and the time was Newton time [10],

$$\frac{d\boldsymbol{P}_{j,f,u,g}}{dt_{\text{Newton}}} = -G_x m_{k,x} m_{j,x} \boldsymbol{f}_{j,k}, \quad x = f, 0, g. \tag{61}$$

$\boldsymbol{P}_{j,f,u,g}$: Momentum of particle j in motion in field of others. G_x : Static gravitation constant as measured at rest in field. $m_{j,x}$: Restmass of particle j in field of others, as measured at rest at particle location in same. t_{Newton} : Newton time.

Since gravitation constant G may depend on location of measurement of G in field, there is ambiguity concerning location and field that the entity is referred to in the equation. Plurality of ways may be devised to interpret the law that shall lead to corresponding outcomes/consequences. One way to address the ambiguity is preserving symmetry of the law in the name of *interaction*. Accordingly,

$$\frac{d\boldsymbol{P}_{j,f,u,g}}{dt_{\text{Newton}}} = -G_{k,x}^{1/2} m_{k,x} G_{j,x}^{1/2} m_{j,x} \boldsymbol{f}_{j,k}. \tag{62}$$

$G_{k,x}$: Static gravitation constant as measured at rest at location of particle k in field of others. $G_{j,x}$: Static gravitation constant as measured at rest at location of particle j in field of others.

Newton time is conditionally equivalent to RT but not available in field environment. It is also unknown if locally defined common time of g-frame would/could exist. While local time in field can exist, e.g., field AT, such time is not differentiable with respect to location in field, except in region of equal potential. Therefore, the only time suitable for equation of motion in field is ST of particle in motion in field. Since SLV is defined on AT, therefore, in compliance, SAT of particle in motion in field shall be used in refining the original LNG. Accordingly, on AT under ISA,

$$\frac{d\boldsymbol{P}_{j,f,u,g}}{dt_{j,s,u,g}} = -G_{k,x}^{1/2} m_{k,x} G_{j,x}^{1/2} m_{j,x} \boldsymbol{f}_{j,k}. \tag{63}$$

$t_{j,s,u,g}$: SAT of particle j in motion in field of others.

Thus, refined LNG reads that alteration of field momentum of particle in SAT of same is proportional to restmass of the particle in field as well as that causing the field and inversely to square of distance between centers of the parties in interaction.

Mass particle in motion in SGF can be viewed as internal motion of a system comprising particle and field (particle/field system, PFS). Therefore, under **Assumption 2**, with Equation (12), (15), and (35),

$$\begin{gathered} \begin{matrix} G_{f,x} = G_i \beta_{f,g}^{2n_g} \\ G_{p,x} = G_i \beta_{p,g}^{2n_g} \end{matrix}, \quad \begin{matrix} M_{f,x} = M_i \\ m_{p,x} = \beta_{p,g}^{-1} m_i \end{matrix}, \quad \begin{matrix} \beta_{f,g} = 1 \\ \beta_{p,g} \equiv \beta_g \end{matrix}, \quad dt_{s,u,g} = \frac{dt_i}{\beta_u \beta_g} \quad \rightarrow \\ \beta_u \frac{d}{dt_i}\left(\frac{\boldsymbol{u}}{\beta_u}\right) = -\frac{G_i M_i}{c_i} \frac{\hat{\boldsymbol{r}}}{\beta_g^{2-n_g} r^2} \end{gathered}. \tag{64}$$

G_i : Gravitation constant as measured in RS. M_i : Restmass responsible for field, as measured in RS. β_g : Schwarzschild Factor of particle in field. β_u : Lorentz Factor of particle in motion. t_i : RAT. m_i : Restmass of particle in RS. $\boldsymbol{u}$: Reduced velocity of particle in motion in field. r: Distance between particle and field center. $\hat{\boldsymbol{r}}$: Unit vector of r.

Denote

$$\begin{gathered} r_g \equiv \frac{G_i M_i}{c_i^2}, \rho \equiv \frac{r}{r_g}, d\tau \equiv \frac{c_i}{r_g} dt_i \quad \rightarrow \\ \beta_u \frac{d}{d\tau} \frac{\boldsymbol{u}}{\beta_u} = -\frac{\hat{\boldsymbol{\rho}}}{\beta_g^{2-n_g} \rho^2}, \ \beta_g = \left(1 - \left(2 - n_g\right) / \rho\right)^{1/\left(2-n_g\right)} \end{gathered}. \tag{65}$$

r_g: CLF of SGF. ρ: Distance between particle and field center, in unit of CLF. τ: Reduced RAT. $\hat{\boldsymbol{\rho}}$:

Unit vector of $\boldsymbol{\rho}$.

Thus, in reduced RF units, refined LNG for PFS under ISA is

$$\boldsymbol{a}-f_u\boldsymbol{u}+f_g\hat{\boldsymbol{\rho}}=\boldsymbol{0},\ \boldsymbol{a}\equiv\frac{d\boldsymbol{u}}{d\tau},\ f_u\equiv\frac{d}{d\tau}\ln\beta_u,\ \beta_u\equiv\sqrt{1-u^2},\ f_g\equiv\beta_g^{n_g-2}\rho^{-2}. \quad (66)$$

Under ISA, motion of particle in field is confined in Euclid plane containing field center, due to conservation nature of centripetal interaction. It is thus convenient to use cylindrical numeral system for coordination. Define

$$\boldsymbol{\rho}\equiv\boldsymbol{i}\rho\cos\theta+\boldsymbol{j}\rho\sin\theta,\ q\equiv\frac{d\rho}{d\tau},\ \omega\equiv\frac{d\theta}{d\tau},\ \chi\equiv\frac{dq}{d\tau},\ \psi\equiv\frac{d\omega}{d\tau}. \quad (67)$$

$\boldsymbol{\rho}$: Reduced location vector of particle in cylindrical coordinates. ρ: Reduced distance between particle and field center assigned as origin. θ: Angular location of particle in cylindrical coordinates. $\boldsymbol{i},\boldsymbol{j}$: Orthogonal unit vectors defining plane of motion containing origin.

Vector Equation (66) is then decomposed into component equations along radius and angular directions,

$$\begin{array}{ccccc} \rho\psi+2\omega q-f_u\rho\omega=0 & \rightarrow & \chi=\rho\omega^2-\left(1-q^2\right)f_g & \rightarrow & \\ \chi-\rho\omega^2-f_uq+f_g=0 & & \psi=q\omega\left(f_g-2/\rho\right) & & \\ & & \dfrac{d}{d\rho}\ln\left[\rho^2\omega\right]=\dfrac{d}{d\rho}\ln\beta_u=f_g & & \end{array}. \quad (68)$$

With Equation (15),

$$\begin{array}{ccccc} d\ln\dfrac{\beta_g}{\beta_u}=0 & & \dfrac{\beta_g}{\beta_u}=\epsilon & & q^2=1-\dfrac{\beta_g^2}{\epsilon^2}\left(1+\dfrac{\gamma^2}{\rho^2}\right) \\ & \rightarrow & & \rightarrow & \\ d\ln\dfrac{\rho^2\omega}{\beta_u}=0 & & \dfrac{\rho^2\omega}{\beta_u}=\gamma & & \omega^2=\dfrac{\gamma^2\beta_g^2}{\epsilon^2\rho^4} \end{array}. \quad (69)$$

ϵ : Integration constant, total energy of particle in field in reduced unit. γ: Integration constant, angular momentum of particle in field in reduced unit.

Therefore,

$$\begin{array}{ccccc} \beta_u=\beta_g/\epsilon,\quad \epsilon\neq 0 & & \lim\limits_{\beta_g\to 0}\beta_u=0 & & \lim\limits_{\beta_g\to 0}u^2=1 \\ & \rightarrow & & \rightarrow & \\ \rho\omega=\gamma\beta_u/\rho,\quad \gamma\neq 0 & & \lim\limits_{\beta_u\to 0}\rho\omega=0 & & \lim\limits_{\beta_u\to 0}q^2=1 \end{array}. \quad (70)$$

That is, if mass particle were to rendezvous onto SS then, at landing site, transverse velocity of the particle shall approach zero and impact velocity approach local SLV, which is zero along surface normal of SS at landing site, while total energy and angular momentum of particle shall remain intact during rendezvous.

For mass particle in orbital motion outside SS of SGF, at apsidal distances,

$$q\big|_{\rho=a,b}=0,\ \rho_{\mathrm{SS}}<b\le a\ \rightarrow\ \epsilon^2=\frac{\left(a^2-b^2\right)\beta_{g,a}^2\beta_{g,b}^2}{a^2\beta_{g,b}^2-b^2\beta_{g,a}^2},\ \gamma^2=\frac{a^2b^2\left(\beta_{g,a}^2-\beta_{g,b}^2\right)}{a^2\beta_{g,b}^2-b^2\beta_{g,a}^2}. \quad (71)$$

a : Maximum distance between orbiting particle and field center in reduced unit. b: Minimum distance between orbiting particle and field center in reduced unit.

That is, orbital energy and angular momentum of mass particle orbiting in field are completely determined by orbital apsides, and circular orbit is of relatively lower ϵ and γ. Further, exists minima in orbital energy of particle in circular motion in field, referred to as ground state,

$$\epsilon_{\mathrm{o}} \equiv \lim_{b \to a} \epsilon = \beta_{g,a}^{2-n_g/2} / \sqrt{1-\left(3-n_g\right)/a} \rightarrow a_{\epsilon_{\mathrm{o}},\min} = 6-2n_g,\ \epsilon_{\mathrm{o},\min} = \sqrt{2}\beta_{g,6-2n_g}^{2-n_g/2}. \quad (72)$$

ϵ_{o} : Total energy of particle in circular orbit in field in reduced unit.

Ground state is also of minimal angular momentum. If particle is circling in vicinity of SS closer than that of ground state then orbital energy of particle shall become higher for such maneuvering that shall approach infinity at orbital radius $\rho_{\mathrm{SS}}+1$, *i.e.*, one CLF away from SS. Therefore, mass particle in circular orbit in vicinity of SS cannot maneuver onto SS even under strong gravitation attraction of SS unless there are channels for dissipation of the energy (hence noncircular motion). On the other hand, total energy of particle in ground state is only marginally lower than that in RS. For instance,

$$\begin{array}{llll} n_g = 0: & \epsilon_{\mathrm{o}} = \left(1-2/a\right)/\sqrt{1-3/a} & a_{\epsilon_{\mathrm{o}},\min} = \rho_{\mathrm{SS}}+4 & \epsilon_{\mathrm{o},\min} = 2\sqrt{2}/3 \\ n_g = -2: & \epsilon_{\mathrm{o}} = \left(1-4/a\right)^{3/4}/\sqrt{1-5/a}, & a_{\epsilon_{\mathrm{o}},\min} = \rho_{\mathrm{SS}}+6, & \epsilon_{\mathrm{o},\min} = \sqrt{2}\left(3/5\right)^{3/4} \end{array}. \quad (73)$$

Therefore, the energy required to dissipate is tremendous for ground state particle maneuvering onto SS.

For particle in motion towards SS along geodesic containing field center,

$$\gamma = 0 \rightarrow \epsilon^2 = \frac{\beta_{g,\rho}^2}{1-q^2} = \frac{\beta_{g,\rho_0}^2}{1-q_0^2} \rightarrow q = -\sqrt{1-\left(1-q_0^2\right)\beta_{g,\rho}^2/\beta_{g,\rho_0}^2}. \quad (74)$$

ρ_0 : Distance of particle to field center whereat velocity of particle is measured. q_0 : Momentary velocity of particle towards field center as measured at ρ_0.

That is, regardless of initial velocity, mass particle shall approach but not reach local SLV until touchdown at SS while total energy of the particle shall remain as invariant during the entire process.

11. Atomic Clock Onboard Satellite

Consider an atomic clock onboard satellite orbiting in Earth field approximated as SGF of spherical symmetry. For particle defining the clock, from Equation (35) and (69), on AT under ISA,

$$\begin{array}{c} \epsilon_{s,o} = \beta_{u,o}\beta_{g,o} = \beta_{g,o}^2/\epsilon_o,\ \beta_{u,o} \equiv \sqrt{1-u_o^2},\ \beta_{g,o} \equiv \beta_{g,\rho_o} \rightarrow \\ \epsilon_o = \beta_{g,a}\beta_{g,b}\sqrt{a^2-b^2}/\sqrt{a^2\beta_{g,b}^2-b^2\beta_{g,a}^2} \\ \Delta E_{s,o} = \beta_{u,o}\beta_{g,o}\Delta E_{s,\boldsymbol{i}} \rightarrow \nu_{o,\boldsymbol{i}} = \dfrac{\beta_{g,\rho_o}^2}{\epsilon_o}\nu_{\boldsymbol{i}} \end{array}. \quad (75)$$

$\epsilon_{s,o}$: Selfenergy of mass particle defining atomic clock onboard satellite, in reduced unit. ϵ_o : Orbital energy of particle in field defining atomic clock onboard satellite, in reduced unit. u_o : Momentary velocity of onboard clock with respect to field, in reduced unit. ρ_o : Momentary distance between satellite and field center, in reduced unit. a,b : Apogee and perigee of satellite orbit, in reduced unit. $\Delta E_{s,o}$: Selfenergy difference of particle defining atomic clock onboard satellite. $\Delta E_{s,\boldsymbol{i}}$: Selfenergy difference of particle defining atomic clock in RS. $\nu_{o,i}$: Frequency of atomic clock onboard satellite, in unit of RAT. ν_i : Frequency of atomic clock in RS, in unit of RAT.

Therefore, ticking rate of atomic clock onboard satellite is a function of momentary distance between the clock and field center.

For clock comparison, an otherwise identical atomic clock at rest on Earth surface is of

$$\nu_{e,i} = \beta_{u,e}\beta_{g,e}\nu_i,\ \beta_{u,e} \equiv \sqrt{1-u_e^2},\ \beta_{g,e} \equiv \beta_{g,\rho_e}. \tag{76}$$

$\nu_{e,i}$: Frequency of stationary on-Earth atomic clock, in unit of RAT. u_e : Momentary velocity of on-Earth clock with respect to field, in reduced unit. ρ_e : Momentary distance between on-Earth clock and field center, in reduced unit.

Thus, relative frequency (ticking rate in identical time unit) difference between the two atomic clocks is

$$\delta \equiv \frac{\Delta\nu}{\nu_{e,i}} \equiv \frac{\nu_{o,i}-\nu_{e,i}}{\nu_{e,i}} = \frac{\beta_{u,o}}{\beta_{u,e}}\frac{\beta_{g,o}}{\beta_{g,e}} - 1 = \frac{\beta_{g,\rho_o}^2}{\epsilon_o\sqrt{1-u_e^2}\,\beta_{g,e}} - 1. \tag{77}$$

For quantitative comparison of atomic clocks, suppose satellite in consideration is in semi-synchronous circular orbit around Earth passing NASA Goddard Space Flight Center (GSFC) twice daily and on-Earth clock at rest on ground level of GSFC. From Equation (72),

$$\frac{2r_g\omega_{e,i}}{c_i} = \frac{\beta_{g,a_o}^{n_g/2-1}}{a_o^{3/2}} \rightarrow \begin{array}{l} a_o\big|_{n_g=0} \approx 5.989\ 085\ 576\times10^9 \\ a_o\big|_{n_g=-2} \approx 5.989\ 085\ 577\times10^9 \end{array}. \tag{78}$$

a_o : Radius of circular orbit of semi-synchronous satellite in reduced unit; latitude of GSFC is 38˚59′48″N, altitude 66 m, as retrieved from Google Earth dated 2018; also assumed spheroid model of Earth for estimation of sea level at site. r_g : CLF of Earth field. $\omega_{e,i}$: Momentary spin velocity of Earth as measured in RF. c_i : SLV as measured/defined on AT in RS.

It is thus difficult to differentiate n_g via orbital radius of the satellite. On the other hand,

$$\begin{gathered} \delta = \frac{\beta_{g,a_o}^{n_g/2}}{\beta_{g,e}}\frac{\sqrt{a_o-3+n_g}}{\sqrt{a_o}\sqrt{1-u_e^2}} - 1 \rightarrow \\ \begin{array}{l} \delta\big|_{n_g=0} \approx 4.470\ 197\ 658\times10^{-10} \\ \delta\big|_{n_g=-2} \approx 4.470\ 197\ 663\times10^{-10} \end{array} \rightarrow \Delta\nu \approx \begin{cases} 38.517\ 051\ 58 \\ 38.517\ 051\ 62 \end{cases} \frac{\mu\text{s}}{\text{SD}} \end{gathered}. \tag{79}$$

That is, atomic clock onboard satellite shall be ticking faster than its counterpart on-Earth. For the atomic clocks in consideration, the on-board one runs faster than corresponding one on-ground by ~38 microseconds per full spin of Earth (sidereal day, SD), as observed in GPS satellites [27]. Alignment of comparison of the clocks may be perceived as:

1) At some time on ground, satellite is seen approaching local zenith;

2) at ground moment satellite is seen near/at local zenith, counts reading of ground clock is recorded, snapshot of satellite taken with respect to sky background, and signal sent to satellite;

3) at satellite moment the signal is received, reading of on-board clock is recorded;

4) by definition, 86 164.090 56 seconds, or 7 920 747 563 × 10^5 tick counts (if clock is based on cesium 133 atom), *i.e.*, one SD, later, satellite shall be seen again in exactly the same direction in sky with respect to exactly the same sky background (orbital motion of Earth is ignored herein per static field approximation);

5) at such ground moment, counts reading of ground clock is recorded and signal of same type sent to satellite in same manner;

6) at satellite moment the signal is received, reading of on-board clock is recorded.

Then, difference of the counts accumulated by on-board clock between pair of events (receiving signal to receiving signal again) shall be more than that of on-ground clock between pair of events (sending signal to sending signal again). For the case in consideration, onboard atomic clock shall accumulate more counts of the ticks, 354 073 ticks per SD if clock is ^{133}Cs based, than the ticks accumulated by on-Earth clock during the events.

However, even if not considering alignment errors, instability of satellite orbit, etc., differentiation of n_g still requires relative precision of the clocks to be better than 10^{-20}, *i.e.*, difference in local simultaneous counting of identical clocks being less than 10 counts per zetta counts accumulated. Therefore, differentiation of n_g via clock comparison is currently unpractical. On the other hand, consistency between the observed time dilation of atomic clock on GPS/Earth and that predicted by PFS model under refined LNG complying with LEM indicates that, to precision of such time measurement, ~10^{-12}, **Assumption 1** is valid for particle in motion in SGF, if time dilation of atomic clock on GPS was indeed measured via protocol similar to that mentioned above.

12. Photon Deflection in Static Gravitation Field

Photon is perceived as particle in this analysis. By LEC, total energy of particle in centripetal force field is invariant to location of same in same. By definition of SLV, photon travels at SLV regardless of presence/absence of field. However, due to time dilation of unit of AT caused by presence of field in vacuo, SLV defined on AT is not field invariant but function of location in field. Consequently, trajectory of photon in field shall not follow geodesic but deviate from it due to alteration of SLV by field hence the phenomenon of photon deflection in field. Proper treatment of the subject requires knowledge on vacuum as propagation media per the law of Hubble-Lemaître [28] as well as interaction between vacuum and electromagnetic field.

In particle model, motion of photon in field is spontaneous/self-driven since interaction between photon and field is none. Therefore, per LEC,

$$dE_{p,s,c,g} = -d\boldsymbol{P}_{p,f,c,g} \cdot \boldsymbol{c}_{p,s,c,g} \rightarrow \frac{dE_{p,s,c,g}}{dt_{p,s,c,g}} = -c_{p,s,c,g}\boldsymbol{u} \cdot \frac{d}{dt_{p,s,c,g}} \frac{E_{p,f,0,g}\boldsymbol{u}}{c_{f,0,g}}. \quad (80)$$

$E_{p,s,c,g}$: Selfenergy of photon in field. $t_{p,s,c,g}$: ST of photon in field. $\boldsymbol{P}_{p,f,c,g}$: Momentum of photon in field. $c_{p,s,c,g}$: SLV as measured by photon in field in ST of photon. $\boldsymbol{u}$: Reduced velocity of photon. $c_{f,0,g}$: SLV in field as measured at rest in field at location of photon. $E_{p,f,0,g}$: Energy of photon in field, *i.e.*, selfenergy increment of photon absorbing particle at rest at location of photon in field.

In analogy to particle in motion in field, Equation (35),

$$\begin{aligned} E_{p,s,c,g} &= \beta_{p,c} E_{p,f,0,g} \\ c_{p,s,c,g} &= \beta_{p,c} \beta_g c_i \end{aligned} \rightarrow \frac{d}{dt_{p,s,c,g}} \beta_{p,c} E_{p,f,0,g} = -\beta_{p,c} \beta_g \boldsymbol{u} \cdot \frac{d}{dt_{p,s,c,g}} \frac{E_{p,f,0,g} \boldsymbol{u}}{\beta_g}. \quad (81)$$

c_i : SLV as measured/defined in RS. $\beta_{p,c}$: Lorentz Factor of photon. β_g : Schwarzschild Factor of particle in field.

From Equation (52), energy of photon in field is invariant to location of same in same,

$$\begin{aligned} &\frac{dE_{p,f,0,g}}{dt_{p,s,c,g}} = 0 \rightarrow \frac{d\beta_{p,c}}{d\beta_g} = \frac{\beta_{p,c}}{\beta_g} \rightarrow \\ &\beta_{p,c} = \beta_g \rightarrow c_s \equiv c_{p,s,c,g} = \beta_g^2 c_i \rightarrow dt_s \equiv \frac{ds}{c_s} = \frac{ds}{\beta_g^2 c_i} \end{aligned}. \quad (82)$$

ds: Line element of photon trajectory in vacuo. dt_s: Differential of ST of photon.

For photon traveling in field between any point a and b outside SS of field,

$$\Delta t_{s,ab} = \int_a^b \frac{ds}{c_s} = \frac{r_g}{c_i} \int_a^b \frac{\sqrt{\rho^2 + \rho'^2}}{\beta_g^2} d\theta, \ \rho' \equiv \frac{d\rho}{d\theta}. \quad (83)$$

$\Delta t_{s,ab}$: ST taken by photon to travel from a to b. θ: Angular location of photon in field centered at origin. ρ: Distance between photon location and origin in reduced unit.

Among all possible paths between a and b, photon shall take the one with extreme ST of photon [29]. Thus,

$$\mathcal{D}\Delta t_{s,ab} = 0, \ \rho > \rho_{\mathrm{SS}} \rightarrow \frac{1}{2}\frac{d\rho'^2}{d\rho} = \left(\frac{\rho - 4 + n_g}{\rho - 2 + n_g}\right)\rho + \left(\frac{\rho - 3 + n_g}{\rho - 2 + n_g}\right)\frac{2\rho'^2}{\rho}. \quad (84)$$

$\mathcal{D}$: Variation operator.

Therefore,

$$\frac{\rho'^2}{\rho^2} = \frac{\gamma\rho^2}{\beta_g^4} - 1, \ \rho'\big|_{\rho=b} = 0 \rightarrow \left(\frac{d\theta}{d\rho}\right)^2 = \frac{1}{\rho^2}\left(\frac{\rho^2}{b^2}\frac{\beta_{g,b}^4}{\beta_{g,\rho}^4} - 1\right)^{-1}. \quad (85)$$

γ: Integration constant. b: Distance between deflection point of photon trajectory and field center, in reduced unit.

By definition,

$$\theta_D \equiv 2\int_b^\infty \left(\frac{d\theta}{d\rho}\right) d\rho - \pi \rightarrow \theta_D = \frac{4}{b} + \frac{(6 - n_g)\pi - 8}{b^2} + \cdots. \quad (86)$$

θ_D: Deflection angle of photon trajectory in field.

The first term in the expression is identical to that from GRT and independent of n_g. Therefore, determination of n_g would have to resort to second term of Expression (86). On the other hand, deflection radius b of photon trajectory in field could be as short as ~10^0, e.g., in vicinity of SBH. Therefore, contribution to photon deflection from high order terms could be significant.

Star pattern formed by light emitting objects (LEOs) at/near internal radius of physical space from observer is essentially invariant with respect to state of motion of Earth, Sun, Galactic System, Virgo Supercluster, etc., in decades/centuries time scale [28]. Therefore, if a LEO at/near equatorial sphere (inferred from Hubble-Lemaître redshift of the LEO) is observed moving against the background of invariant star pattern with respect to observer in direction opposite to that of observer, then it is plausible that the observed light from the LEO was deflected by optical inactive object near the light path. Therefore, from correlation between angular motion of such LEO (with respect to observer) and location of observation in plane of orbital motion of observer, it is possible to extract the photon deflection function hence determining n_g therefrom.

Nowadays, photon deflection in field, known also as gravitation lens [6], is a fact of observation established beyond reasonable doubt, regardless of theory/interpretation. Therefore, the mere fact of photon deflection in field demonstrates that SLV in field is not field invariant but function of location in field regardless of the time basis the SLV is defined thereupon. In other words, field in vacuo does alter property of vacuo in field hence SLV therein, whether or not SLV is defined on AT.

13. Anomalous Precession of Particle Motion in Field

Anomalous precession (AP) of PFS refers to the phenomenon that, in absence of external influence of any kind, angular position (with respect to g-frame with origin at field center) of orbiting particle at extreme distance to origin is not stationary but advancing with orbital evolution. Accordingly, if a straight line connects particle and field center at the moment distance between particle and origin is at extreme then, upon orbiting, consecutive lines of same shall exhibit angular progression with respect to g-frame. Such progression was not derivable from original LNG hence *anomalous* but predicted by GRT [7] and predictable with refined LNG, Equation (63).

By definition of Expression (67),

$$\begin{gathered} q \equiv \frac{d\rho}{d\tau},\ \omega = q\frac{d\theta}{d\rho} \ \rightarrow\ \frac{d\tau}{d\rho} = \frac{1}{q},\ \frac{d\theta}{d\rho} = \frac{\omega}{q} \ \rightarrow \\ T_{AP} \equiv 2\int_b^a \frac{d\rho}{q},\ \theta_{AP} \equiv 2\int_b^a \frac{\omega}{q} d\rho - 2\pi \end{gathered} . \tag{87}$$

T_{AP}: Apsidal orbital period (AOP), duration from moment at apside to next at same, in reduced RAT. θ_{AP}: AP of particle in orbital motion in field per AOP. a : Maximal distance between orbiting particle and field center, in reduced unit. b : Minimal distance between orbiting particle and field center, in reduced unit.

From Expression (69) and (71),

$$\frac{1}{\sqrt{q^2}}=\left(1-\frac{a^2-\rho^2}{a^2-b^2}\frac{b^2\beta_{g,\rho}^2}{\rho^2\beta_{g,b}^2}-\frac{\rho^2-b^2}{a^2-b^2}\frac{a^2\beta_{g,\rho}^2}{\rho^2\beta_{g,a}^2}\right)^{-1/2}$$
$$\sqrt{\frac{\omega^2}{q^2}}=\frac{1}{\rho}\left(\frac{\beta_{g,a}^2-\beta_{g,\rho}^2}{\beta_{g,a}^2-\beta_{g,b}^2}\frac{\rho^2\beta_{g,b}^2}{b^2B_\rho}+\frac{\beta_{g,\rho}^2-\beta_{g,b}^2}{\beta_{g,a}^2-\beta_{g,b}^2}\frac{\rho^2\beta_{g,a}^2}{a^2\beta_{g,\rho}^2}-1\right)^{-1/2}. \quad (88)$$

Thus, for PFS,

$$T_{AP,\text{PFS}}=\frac{\pi(a+b)^{3/2}}{\sqrt{2}}\left(1+\frac{2}{a+b}+\cdots\right),\ \theta_{AP,\text{PFS}}=\pi\left(2-\frac{n_g}{2}\right)\left(\frac{1}{a}+\frac{1}{b}\right)+\cdots. \quad (89)$$

Therefore, AP is an intrinsic property of particle in orbital motion in field. Such can be understood as caused by selfenergy reduction of particle in field hence selfmass increasing of same in same (with respect to that in RS) by LME hence enhanced particle/field interaction by LNG. Consequently, AP is essentially in inverse proportion to distance between particle and field center and always positive, *i.e.*, always advancing beyond 2π in one AOP. AOP is defined on apsides of particle orbit hence self-contained within the system, which differs from sidereal orbital period (SOP). The latter is defined as time taken for orbiting particle to evolve 2π with respect to *g*-frame or sky background in practice. Expression for SOP is identical to the first term of AOP in Expression (89). Therefore, AOP is always longer than SOP since second term in the expression is always positive.

While the phenomenon is minuscule, Expression (89) indicates that AP is sensitive to n_g. Therefore, measurement of AP of PFS in physical world may be utilized to determine the parameter n_g in **Assumption 2**. Accordingly, regard Mercury as mass particle and Sun as source of SGF and compare such PFS with apsidal precession of Mercury around Sun (APM) [30]. The best fit is found corresponding to $n_g = -2$. Therefore, Expression (89) is revised to

$$T_{AP,\text{PFS}}=\frac{\pi(a+b)^{3/2}}{\sqrt{2}}\left(1+\frac{2}{a+b}+\cdots\right)$$
$$\theta_{AP,\text{PFS}}=3\pi\left(\frac{1}{a}+\frac{1}{b}\right)+\frac{51\pi}{4}\left(\frac{1}{a}+\frac{1}{b}\right)^2+\cdots. \quad (90)$$

Thus, by APM measurement, **Assumption 2** is revised to: static gravitation constant is state function of SGF in form

$$G_{f,0,g}=\beta_g^{-4}G_i. \quad (91)$$

Therefore, static gravitation constant is indeed not genuine physical constant but function of location of measurement in field. Further, from the expression, if G is measured in vicinity of SS then the attribute shall be approaching infinity.

The first term of the expression for AP of PFS in Expression (90) is identical to that from GRT. However, the assumption of $n_g = 0$ was implied in GRT that should have led to AP prediction of GRT to 2/3 of that of Expression (90) nor would such had been in line with observed APM, suggesting existence of LEC violation or internal inconsistency or combination thereof in GRT framework.

Aside from APM observation, Expression (90) may also be falsified via, e.g., observation of passive satellite orbiting around Earth, since mass of satellite is much lighter than that of Earth hence satellite/Earth can be approximated as PFS, assuming spherical SGF of Earth, negligible interference from Moon/Sun/solar wind, clear path of satellite, ignorable Earth motion around Sun, etc. For instance, if a synchronous satellite is of the apsides

$$\begin{matrix} \text{Apogee} \approx 6\ 371 + 70\ 600 \text{ km} \\ \text{Perigee} \approx 6\ 371 + 1\ 000 \text{ km} \end{matrix} \rightarrow \begin{matrix} T_{AP} \approx 86\ 185 \text{ s} \\ \theta_{AP} \approx 0.469 \text{ arcsec JY}^{-1} \end{matrix}. \tag{92}$$

JY: Julian Year, 365.25 JD.

Therefore, by observing configuration of the PFS with respect to ground observer at moment of, e.g., minimum distance between centers of satellite/Earth field and tracking such for one JY then AP of the satellite should be accumulated to ~0.47″, which is measurable with reasonable effort. If $n_g = 0$ then the accumulated AP should be ~0.32″ and difference of such magnitude should be significant enough to falsify Expression (91).

Complication of AP measurement of satellite/Earth system lies in the real-time determination of Earth-satellite distance. In principle, momentary distance between satellite and Earth may be measured by, e.g., sending photon from Earth observatory towards satellite and observing arrival time of the reflected photon therefrom. Assuming synchronous satellite is essentially stationary at local zenith of observatory, ignoring effect of fields of Moon/Sun, then, one-way AT delay of the photon is

$$\Delta t_{i,\rho_E \to \rho_s} = \frac{r_g}{c_i} \int_{\rho_E}^{\rho_s} \frac{ds}{\beta_{g,E}} = \frac{r_g}{c_i}\left(\rho_s - \rho_E\right) + \frac{r_g}{c_i}\left(\ln\frac{\rho_s}{\rho_E} + \frac{5}{2}\left(\frac{1}{\rho_E} - \frac{1}{\rho_s}\right) + \cdots\right). \tag{93}$$

Δt_i : RAT duration for photon to travel from observatory to reflector of satellite. ρ_E : Distance between emission point of photon and center of Earth field, in reduced unit. ρ_s : Distance between center of Earth field and satellite reflector at local moment of photon reflection, in reduced unit. $\beta_{g,E}$: Schwarzschild Factor at location of photon in field of PFS. *ds*: Line element of photon trajectory in field, in reduced unit.

The terms after the first one in Expression (93) are due to non-constancy of SLV in field, also known as Shapiro delay [31] under the configuration and approximations specified. It is therefore not straightforward to extract distance information from RAT delay of photon signal without *a prior* knowledge of the distance being probed. Further, there is no direct access of RAT on Earth having sufficient time resolution, and atomic clock at observatory (station clock) is not identical or equivalent to RAT but function of state of motion and field environment of the station. Therefore, duration of unit of station clock shall have periodic variation twice daily due to tidal changes of mass distribution on Earth, diurnal oscillation caused by variations of distances between station and Moon/Sun, parallel/anti-parallel spin velocity of station with respect to orbital motion of Earth, etc., unless station clock were to locate at spin pole of Earth. In such case, however, synchronous satellite shall not appear as if fixed at zenith of observatory, Shapiro delay become more complicate due to non-geodesic photon path, atmospheric interference become stronger, etc., even if effect of fields of Moon/Sun is ignored.

In addition, there is annual variation of duration of unit of station time regardless of location of station on Earth. If Earth/Sun system is approximated as PFS then

$$\epsilon_E = \beta_g \beta_u,\ \epsilon_f = \beta_g / \beta_u \ \rightarrow\ \epsilon_E = \beta_g^2 / \epsilon_f \ \rightarrow\ \mathbb{U}_{\mathrm{AT},E} = \epsilon_f \beta_g^{-2} \mathbb{U}_{\mathrm{AT},\boldsymbol{i}}. \tag{94}$$

ϵ_E : Selfenergy of particle of PFS, in reduced unit. β_g : Schwarzschild Factor of particle in field of PFS. ϵ_f : Orbital energy of particle of PFS in field of PFS, in reduced unit. β_u : Lorentz Factor of particle of PFS. $\mathbb{U}_{\mathrm{AT},E}$: Unit of SAT of particle of PFS. $\mathbb{U}_{\mathrm{AT},\boldsymbol{i}}$: Unit of RAT.

That is, duration of SI second on Earth has seasonal variations proportional to reduced orbital energy of Earth, which is invariant, and inversely to β_g^2, which is function of distance between Earth and Sun. Therefore, such variation is annual periodic with respect to seasons in year due to elliptic shape of Earth orbit around Sun. Accordingly, duration of events as measured with SI time on Earth shall exhibit same seasonal variations but in inverse manner. In other words, Earth/Sun distance and angular advance rate of perihelion of Earth shall exhibit annual periodicity if the measurement is based on SI time on Earth.

14. Gravitation Constant

Gravitation constant is commonly expressed as [32]

$$\begin{gathered} G \equiv 6.674\ 28 \times 10^{-11} \pm 6.7 \times 10^{-15}\ \mathrm{m}^3\ \mathrm{kg}^{-1}\ \mathrm{s}^{-2} \ \rightarrow \\ G \equiv n_G\ \mathbb{U}_{L,\mathrm{SI}}^3 \mathbb{U}_{m,\mathrm{SI}}^{-1} \mathbb{U}_{t,\mathrm{SI}}^{-2},\ n_G \approx 6.674\ 28(67) \times 10^{-11} \end{gathered}. \tag{95}$$

G: Gravitation constant. n_G : Numeral aspect of G. $\mathbb{U}_L$: Unit of length, meter in SI. $\mathbb{U}_{\mathrm{AT}}$: Unit of AT, second in SI. $\mathbb{U}_m$: Unit of mass, kilogram in SI.

State of units of G is unspecified in the expression. If all the units involved in G in Expression (95) were understood as of in-situ type then, by ROM,

$$G_x = n_G \mathbb{U}_{G,x},\ \mathbb{U}_{G,x} \equiv \mathbb{U}_{L,x}^3 \mathbb{U}_{m,x}^{-1} \mathbb{U}_{\mathrm{AT},x}^{-2} \ \rightarrow\ G_x / G_{\boldsymbol{i}} = \beta_x^3. \tag{96}$$

x : State indicator. β_x : State function. G_i : Gravitation constant as measured in RS free of any field.

That is, G would be state variant but the state dependency does not comply with that from, e.g., APM measurement. Therefore, gravitation constant G in literature has to be understood as expressed in $\boldsymbol{i}$-units, even though that was not originally intended. Thus, under **Assumption 2**, Expression (91),

$$G_x \equiv n_{G,x} \mathbb{U}_{G,\boldsymbol{i}},\ G_{\boldsymbol{i}} \equiv n_{G,\boldsymbol{i}} \mathbb{U}_{G,\boldsymbol{i}} \ \rightarrow\ G_x / G_{\boldsymbol{i}} = n_{G,x} / n_{G,\boldsymbol{i}} = \beta_g^{-4} \ \rightarrow\ G_x = \beta_g^{-4} G_{\boldsymbol{i}}. \tag{97}$$

Accordingly, Equation (63) is rewritten as, under ISA,

$$\frac{d\boldsymbol{P}_{j,f,u,g}}{dt_{j,s,u,g}} = \boldsymbol{F}_{j,k} = -G_i \left(\frac{m_{k,x}}{\beta_{k,g}^2}\right)\left(\frac{m_{j,x}}{\beta_{j,g}^2}\right) \frac{\boldsymbol{r}_j - \boldsymbol{r}_k}{\left\|\boldsymbol{r}_j - \boldsymbol{r}_k\right\|^3},\ \beta_{y,g} \equiv \frac{c_{y,x}}{c_i},\ \begin{matrix} x = f, 0, g \\ y \in \{j, k\} \end{matrix} \tag{98}$$

$\boldsymbol{F}_{j,k}$: Gravitation effect of particle k to particle j. $m_{j,x}$: Restmass of particle j in field of others as measured at rest at location of same in same. $c_{j,x}$: SLV as measured at rest at location of particle j in field of others on AT. $\beta_{j,g}$: Schwarzschild Factor of particle j in field of others. $\boldsymbol{r}_j$: Location vector of particle *j*. c_i : SLV as measured in RS on AT.

By LME,

$$\epsilon_{j,x} = \mu_{j,x}\beta_{j,x}^2 \rightarrow \frac{m_{j,x}}{\beta_{j,g}^2} = \frac{\mu_{j,x}}{\epsilon_{j,x}} m_{j,x} \equiv m_{j,x}^* . \tag{99}$$

$\epsilon_{j,x}$: Reduced restenergy of particle *j* in field of others as measured at rest at location of same in same. $\mu_{j,x}$: Reduced restmass of particle *j* in field of others as measured at rest at location of same in same. $m_{j,x}^*$: Restmass parameter of particle *j* in field of others as measured at rest at location of same in same.

That is, restmass parameter of particle is composite of restmass of particle and ratio of reduced restmass/energy of same. Equation (98) is then expressed as

$$\boldsymbol{F}_{j,k} = -\frac{G_{\boldsymbol{i}} m_{j,x}^* m_{k,x}^*}{r_{j,k}^2} \hat{\boldsymbol{r}}_{j,k} . \tag{100}$$

Therefore, gravitation interaction between particles is not proportional to restmass of particle but restmass parameter of same.

There are other mass parameters in category of astronomical constants [32], e.g., solar mass parameter, geocentric gravitational constant, etc. Such mass parameters are composite of G and mass (presumably restmass) of object not defined on AT but on astronomical event. In essence, astronomical time is built on basis of some recurring astronomical events but interpolated with AT for finer resolution of the time. While ideal astronomical time should be RAT compatible, at least in duration scale of decade/century, such is unrealizable in practice. Therefore, astronomical mass parameters are not truly compatible with corresponding attributes in refined LNG. From Expression (97),

$$(Gm)_x / (Gm)_{\boldsymbol{i}} = \beta_g^{-5} . \tag{101}$$

That is, on AT basis, astronomical mass parameter is not SIT but function of state of object in association.

In essence, astronomical mass parameters were obtained from fitting of trajectories of solar objects with models built on original LNG and/or GRT modification thereof, wherein, G was assumed *a prior* as genuine constant and restmass of object SIT. However, G could be understood as genuine constant but only if restmass of object is replaced by restmass parameter of same, as in Equation (100). Further, restmass and parameter thereof is not SIT by LME. Therefore, ephemeris built on such entities with original LNG/GRT modification is fundamentally different from that with refined LNG, even if ephemerides from different models may be of identical appearance.

15. Apsidal Precession of Binary Particle System

Consider a binary particle system (BPS) at rest in RF. From Equation (98), under ISA,

$$\boldsymbol{F}_1 = -G_{\boldsymbol{i}} \frac{m_{1,x}}{\beta_{1,g}^2} \frac{m_{2,x}}{\beta_{2,g}^2} \frac{\boldsymbol{r}_1 - \boldsymbol{r}_2}{\|\boldsymbol{r}_1 - \boldsymbol{r}_2\|^3}, \boldsymbol{F}_2 = -G_{\boldsymbol{i}} \frac{m_{2,x}}{\beta_{2,g}^2} \frac{m_{1,x}}{\beta_{1,g}^2} \frac{\boldsymbol{r}_2 - \boldsymbol{r}_1}{\|\boldsymbol{r}_2 - \boldsymbol{r}_1\|^3}, x = f, 0, g . \tag{102}$$

By LEC,

$$dw = -\boldsymbol{F}\cdot d\boldsymbol{r} \rightarrow \begin{matrix} dE_{1,x} = G_i \dfrac{m_{1,x}}{\beta_{1,g}^2}\dfrac{m_{2,x}}{\beta_{2,g}^2}\dfrac{(\boldsymbol{r}_1-\boldsymbol{r}_2)\cdot d\boldsymbol{r}_1}{\|\boldsymbol{r}_1-\boldsymbol{r}_2\|^3} \\ dE_{2,x} = G_i \dfrac{m_{2,x}}{\beta_{2,g}^2}\dfrac{m_{1,x}}{\beta_{1,g}^2}\dfrac{(\boldsymbol{r}_2-\boldsymbol{r}_1)\cdot d\boldsymbol{r}_2}{\|\boldsymbol{r}_2-\boldsymbol{r}_1\|^3} \end{matrix}. \tag{103}$$

dw_j: Work done to particle *j* for infinitesimal displacement/velocity in field of the other. $E_{j,x}$: Restenergy of particle *j* in field of the other.

By LME,

$$\begin{matrix} dE_{1,x} = E_{1,x}\dfrac{G_i m_{2,x}(\boldsymbol{r}_1-\boldsymbol{r}_2)\cdot d\boldsymbol{r}_1}{c_i^2\beta_{1,g}^4\beta_{2,g}^2\|\boldsymbol{r}_1-\boldsymbol{r}_2\|^3} \\ dE_{2,x} = E_{2,x}\dfrac{G_i m_{1,x}(\boldsymbol{r}_2-\boldsymbol{r}_1)\cdot d\boldsymbol{r}_2}{c_i^2\beta_{2,g}^4\beta_{1,g}^2\|\boldsymbol{r}_2-\boldsymbol{r}_1\|^3} \end{matrix} \rightarrow \begin{matrix} d\Delta E_{1,x} = \Delta E_{1,x}\dfrac{G_i m_{2,x}(\boldsymbol{r}_1-\boldsymbol{r}_2)\cdot d\boldsymbol{r}_1}{c_i^2\beta_{1,g}^4\beta_{2,g}^2\|\boldsymbol{r}_1-\boldsymbol{r}_2\|^3} \\ d\Delta E_{2,x} = \Delta E_{2,x}\dfrac{G_i m_{1,x}(\boldsymbol{r}_2-\boldsymbol{r}_1)\cdot d\boldsymbol{r}_2}{c_i^2\beta_{2,g}^4\beta_{1,g}^2\|\boldsymbol{r}_2-\boldsymbol{r}_1\|^3} \end{matrix}. \tag{104}$$

$\Delta E_{j,x}$: Selfenergy difference of particle defining SAT of particle *j* in field of the other.

By definition of unit of AT and SLV defined on AT,

$$\Delta E_{j,x} = \frac{\mathcal{N}_t h_i c_i}{\mathcal{N}_c \mathbb{U}_L}\beta_{j,g} \rightarrow \begin{matrix} d\beta_{1,g} = \dfrac{G_i m_{2,x}(\boldsymbol{r}_1-\boldsymbol{r}_2)\cdot d\boldsymbol{r}_1}{c_i^2\beta_{1,g}^3\beta_{2,g}^2\|\boldsymbol{r}_1-\boldsymbol{r}_2\|^3} \\ d\beta_{2,g} = \dfrac{G_i m_{1,x}(\boldsymbol{r}_2-\boldsymbol{r}_1)\cdot d\boldsymbol{r}_2}{c_i^2\beta_{2,g}^3\beta_{1,g}^2\|\boldsymbol{r}_2-\boldsymbol{r}_1\|^3} \end{matrix}. \tag{105}$$

With Equation (104),

$$\frac{dE_{j,x}}{d\beta_{j,g}} = \frac{E_{j,x}}{\beta_{j,g}} \rightarrow \frac{E_{j,x}}{E_{j,i}} = \frac{m_{j,i}}{m_{j,x}} = \beta_{j,g},\ j = 1,2. \tag{106}$$

$E_{j,i}$: Restenergy of particle *j* as measured in RS. $m_{j,i}$: Restmass of particle *j* as measured in RS.

Therefore,

$$d\beta_{1,g} = \frac{G_i m_{2,i}(\boldsymbol{r}_1-\boldsymbol{r}_2)\cdot d\boldsymbol{r}_1}{c_i^2\beta_{1,g}^3\beta_{2,g}^3\|\boldsymbol{r}_1-\boldsymbol{r}_2\|^3},\ d\beta_{2,g} = \frac{G_i m_{1,i}(\boldsymbol{r}_2-\boldsymbol{r}_1)\cdot d\boldsymbol{r}_2}{c_i^2\beta_{1,g}^3\beta_{2,g}^3\|\boldsymbol{r}_1-\boldsymbol{r}_2\|^3}. \tag{107}$$

Denote

$$\begin{matrix} \mu_1 \equiv \dfrac{m_{1,i}}{m_{1,i}+m_{2,i}},\ \mu_2 \equiv \dfrac{m_{2,i}}{m_{1,i}+m_{2,i}},\ r_g \equiv \dfrac{G_i}{c_i^2}(m_{1,i}+m_{2,i}),\ \begin{matrix} \boldsymbol{\rho}_j \equiv \boldsymbol{r}_j / r_g \\ \boldsymbol{s} \equiv \boldsymbol{\rho}_1 - \boldsymbol{\rho}_2 \end{matrix} \rightarrow \\ d\beta_{1,g} = \dfrac{\mu_2 \boldsymbol{s}\cdot d\boldsymbol{\rho}_1}{\beta_{1,g}^3\beta_{2,g}^3 s^3},\ d\beta_{2,g} = -\dfrac{\mu_1 \boldsymbol{s}\cdot d\boldsymbol{\rho}_2}{\beta_{2,g}^3\beta_{1,g}^3 s^3} \rightarrow \dfrac{d}{ds}(\mu_1\beta_{1,g}+\mu_2\beta_{2,g}) = \dfrac{\mu_1\mu_2}{\beta_{1,g}^3\beta_{2,g}^3 s^2} \end{matrix}. \tag{108}$$

$\boldsymbol{\rho}_j$: Location vector of particle *j* in reduced unit. ***s***: Distance vector between particles of BPS in reduced unit.

Set origin of RF to center of RS restmass of BPS and take the approximation that

$$\begin{matrix} \begin{matrix} \boldsymbol{\rho}_1 \simeq +\mu_2 \boldsymbol{s} \\ \boldsymbol{\rho}_2 \simeq -\mu_1 \boldsymbol{s} \end{matrix} \rightarrow \dfrac{d\beta_{1,g}}{ds} = \dfrac{\mu_2^2}{\beta_{1,g}^3\beta_{2,g}^3 s^2},\ \dfrac{d\beta_{2,g}}{ds} = \dfrac{\mu_1^2}{\beta_{2,g}^3\beta_{1,g}^3 s^2} \rightarrow \\ \dfrac{d}{ds}(\mu_1^2\beta_{1,g} - \mu_2^2\beta_{2,g}) = 0 \rightarrow \mu_2^2(1-\beta_{2,g}) = \mu_1^2(1-\beta_{1,g}) \end{matrix}. \tag{109}$$

Solution of Equation (108) is then

$$\frac{\mu_2^8}{s}=\frac{1}{4}\delta^3 f_4+\frac{3}{5}\delta^2\mu_1^2 f_5+\frac{1}{2}\delta\mu_1^4 f_6+\frac{1}{7}\mu_1^6 f_7,\quad \begin{array}{l}\delta\equiv\mu_2-\mu_1\\ f_k\equiv 1-\beta_{1,g}^k\end{array},\ 0\le\mu_1\le\frac{1}{2}. \tag{110}$$

Therefore,

$$s_{\mathrm{SD}}\equiv s\big|_{\beta_{1,g}=0}=\frac{\mu_2^8}{\frac{1}{4}\delta^3+\frac{3}{5}\delta^2\mu_1^2+\frac{3}{6}\delta\mu_1^4+\frac{1}{7}\mu_1^6}\rightarrow \begin{array}{l} s_{\mathrm{SD,PFS}}\equiv s_{\mathrm{SD}}\big|_{\mu_1=0}=4\\ s_{\mathrm{SD,SBP}}\equiv s_{\mathrm{SD}}\big|_{\mu_1=\frac{1}{2}}=\frac{7}{4}\end{array}. \tag{111}$$

s_{SD}: Schwarzschild Distance between particles of BPS in reduced unit.

That is, Schwarzschild Distance between particles of PFS is 4 CLFs, *i.e.*, heavier member of PFS shall appear to lighter one of same as black hole of SS radius of 4 units, and that of symmetric binary particle system (SBP) is 7/4, *i.e.*, each member of SBP shall appear to the other of same as black hole of SS radius of 7/8 units, therefore members of SBP are not particle to each other but extended object instead.

From Equation (102), with Equation (109),

$$\begin{array}{l}\boldsymbol{a}_1-f_u\boldsymbol{u}_1+f_g\hat{\boldsymbol{s}}=\boldsymbol{0}\\ f_g\equiv\dfrac{\mu_2}{\beta_{1,g}^4\beta_{2,g}^3 s^2}\end{array}\rightarrow \begin{array}{l} d\ln\dfrac{\beta_{1,g}}{\beta_{1,u}}=0\\ d\ln\dfrac{\rho_1^2\omega_1}{\beta_{1,u}}=0\end{array}\rightarrow \begin{array}{l}\dfrac{\beta_{1,g}}{\beta_{1,u}}=\epsilon_1\\ \dfrac{\rho_1^2\omega_1}{\beta_{1,u}}=\gamma_1\end{array}\rightarrow \begin{array}{l}\omega_1^2=\dfrac{\gamma_1^2\beta_{1,g}^2}{\epsilon_1^2\rho_1^4}\\ q_1^2=1-\dfrac{\beta_{1,g}^2}{\epsilon_1^2}\left(1+\dfrac{\gamma_1^2}{\rho_1^2}\right)\end{array}. \tag{112}$$

ϵ_1: Total energy of lighter member of BPS in motion in field of the other. γ_1: Angular momentum of lighter member of BPS in motion in field of the other.

Under apsis condition,

$$\begin{array}{c} q_1\big|_{\rho_1=\mu_2 a,\mu_2 b}=0,\ s_{\mathrm{SD}}<b\le a\ \rightarrow\\ \epsilon_1^2=\dfrac{\left(a^2-b^2\right)\beta_{1,g,\mu_2 a}^2\beta_{1,g,\mu_2 b}^2}{a^2\beta_{1,g,\mu_2 b}^2-b^2\beta_{1,g,\mu_2 a}^2},\ \gamma_1^2=\dfrac{\mu_2^2 a^2 b^2\left(\beta_{1,g,\mu_2 a}^2-\beta_{1,g,\mu_2 b}^2\right)}{a^2\beta_{1,g,\mu_2 b}^2-b^2\beta_{1,g,\mu_2 a}^2}\end{array}. \tag{113}$$

a: Maximum distance between members of BPS in reduced unit. b: Minimum distance between members of BPS in reduced unit.

Accordingly,

$$\begin{array}{l} T_{AP,\mathrm{BPS}}=\dfrac{\pi\left(a+b\right)^{3/2}}{\sqrt{2}}\left(1+\dfrac{2\left(1-\mu_1\right)^2}{a+b}+\cdots\right)\\ \theta_{AP,\mathrm{BPS}}=3\pi\left(1-2\mu_1+\dfrac{3}{2}\mu_1^2\right)\left(\dfrac{1}{a}+\dfrac{1}{b}\right)+\cdots\end{array}. \tag{114}$$

Therefore,

$$T_{AP,\mathrm{SBP}}=\frac{\pi\left(a+b\right)^{3/2}}{\sqrt{2}}\left(1+\frac{1}{2\left(a+b\right)}+\cdots\right),\ \theta_{AP,\mathrm{SBP}}=\frac{9\pi}{8}\left(\frac{1}{a}+\frac{1}{b}\right)+\cdots. \tag{115}$$

That is, AP of BPS is less than that of PFS, and AP of SBP is the least.

Equation (114) is modelisticly accurate in extreme cases, *i.e.*, PFS ($\mu_1=0$) and SBP ($\mu_1=1/2$), but not exact for BPS inbetween the extremes, due to nonstationary nature of RAT simultaneous restmass center of general BPS. Analysis herein also ignored retardation of SF to mass particle in motion. Nevertheless, field retardation should only cause reduction of AP of BPS, if any, instead of the other way around. The analysis also did not consider effect of field delay, which is caused by motion of field-causing particle and finite speed of field propagation. In principle, such delay would affect AP of BPS which, if any, should be of positive correlation with velocity of particle in motion hence inversely to orbital radii of BPS. On the other hand, among the cases considered herein, AP of PFS is maximal while field of PFS is static hence no field delay therein. Therefore, AP as expressed in Expression (114) with $\mu_1 = 0$ is the up limit for AP of general BPS at rest in RF.

If BPS is noncompact, *i.e.*, separation of members of BPS is sufficiently larger than Schwarzschild Distance of the system, then from Equation (112), with Expression (113),

$$\begin{aligned} s &\simeq \frac{2ab}{a+b+(a-b)\cos\left[(1-\delta)(\theta-\theta_b)\right]} \\ \delta &\equiv \frac{3}{2}\left(1-2\mu_1+\frac{3}{2}\mu_1^2\right)\left(\frac{1}{a}+\frac{1}{b}\right)+\cdots > 0 \end{aligned} . \tag{116}$$

θ_b : Planar angle of BPS at configuration of minimum distance between members of same.

That is, orbit of noncompact BPS is of Keplerian type but with a scaling factor to angle variable θ that is always less than one. By definition of Keplerian orbital parameters,

$$a = \alpha(1+e),\ b = \alpha(1-e)\ \rightarrow\ s = \alpha\left(1-e^2\right)/\left(1+e\cos\left[(1-\delta)(\theta-\theta_b)\right]\right). \tag{117}$$

α: Semi major axis of Keplerian orbit in reduced unit. e: Eccentricity of Keplerian orbit.

Excluding that due to approximation of mass center of Equation (109), the error term of Equation (117) on offset of θ is less than $5b^{-2}/2$ hence the expression is suitable for noncompact BPS at rest in RF.

16. Apsidal Precession of Object in Solar System

For computing AP of object in Solar System, assume object orbiting around Sun can be approximated as BPS at rest in RF, *i.e.*, regarding both objects as mass particles hence omitting effects due to spacial profile of the objects and ignoring interactions of the system with other objects in Solar System and motion of the BPS with respect to sky background. For such BPS,

$$a = \frac{(1+e)\alpha}{r_g},\ b = \frac{(1-e)\alpha}{r_g},\ r_g \equiv \left(1+\frac{1}{\eta}\right)\frac{G_i M_i}{c_i^2},\ \eta \equiv \frac{M_i}{m_i},\ \mu \equiv \frac{1}{1+\eta}. \tag{118}$$

a : Aphelion in reduced unit. b : Perihelion in reduced unit. e : Eccentricity of particle orbit around Sun. α: Semi major axis of particle orbit around Sun. r_g : CLF of BPS. $G_i M_i$: Solar mass parameter as measured in RS. c_i : SLV as measured/defined in RS on RAT. M_i : Restmass of Sun as measured in RS. m_i : Restmass of object as measured in RS.

From Expression (115),

$$\begin{aligned} &\mathrm{SOP} = \frac{2\pi\alpha^{3/2}}{86400\sqrt{G_i M_i + G_i m_i}}\ \mathrm{JD} \\ &\mathrm{AOP} \simeq \left(1 + \frac{2(1-\mu)^2}{a+b}\right)\mathrm{SOP} \end{aligned}, \quad \begin{aligned} &\mathrm{AP} \simeq \left(1 - 2\mu + \frac{3\mu^2}{2}\right) f_r \ \ \mathrm{arcsec.\,JC}^{-1} \\ &f_r \equiv \frac{36525 \times 3 \times 180 \times 3600}{\mathrm{AOP}}\left(\frac{1}{a} + \frac{1}{b}\right) \end{aligned}. \quad (119)$$

With the parameters listed in **Table 2**, AP of some objects in Solar System was calculated, as listed in **Table 3**.

From Expression (119), uncertainty of the calculated SOP came from that of semi major axis and the mass parameters. Uncertainty of α for Planet was unspecified in the reference but may be assumed as ~10^{-8} absolute in *au*. Uncertainty of solar mass parameter is 1×10^{10} absolute [32]. Since restmass of solar object is much lighter than that of Sun, contribution from such object, including those of unknown mass, to uncertainty of sum of the mass parameters is relatively smaller. Replacement of RS restmass with infield restmass shall also introduce systematic error but should be less than 10^{-10} relative. Therefore, uncertainty of the calculated SOP is dominated by that of α but should not be less than 10^{-10} relative. Since objects considered herein are all in far field of Sun, calculated SOP and AOP are statistically indifferent.

From **Table 3**, discrepancy exists between calculated and observed SOPs of the Planets, which is far beyond uncertainties of the associated entities, especially that of the outer Planets. Such may be explained as caused by interactions among the objects. If so, however, such interaction should also be observed in that of Pluto, Ceres, and the asteroids instead of the more than excellent matches between the calculated and observed SOPs.

Recall that Le Verrier's estimation of APM was 38″ per century, which was based on observations accumulated over ~150 years on Mercury transit over Sun disk [30]. Newcomb examined essentially all celestial observations available by his time and estimated APM as ~43″/JC [40]. In either case, APM was contrasted against the background of parametric celestial mechanical model on original LNG or ephemerides derived therefrom. Further, on AOP basis, AP of Mercury is ~0.10″, which is not significantly larger than that of other inner Planets, e.g., ~0.05″ for Venus, ~0.04″ for Earth, ~0.03″ for Mars. However, AP estimations of Newcomb for such Planets were quite off. Still further, Mercury perihelion advancement as observed was/is at least one order of magnitude larger than APM

Table 2. Parameters for AP of Object in Solar System.

Item	Symbol	Parameter
Solar mass parameter	GM_S	1.327 124 400 4 × 10^{20} m^3 s^{-2}[a]
Astronomical Unit	*au*	149 597 870 700 m[a]
Julian Day	JD	86 400 s[b]
Julian Year	JY	365.25 JD
Julian Century	JC	100 JY

[a][32], TDB-compatible. [b]SI second defined in RS.

Table 3. Anomalous Precession of Object in Solar System.

Object	η[a]	a (au)	e	$SOP_{Cal.}$ (JD)	$SOP_{Obs.}$ (JD)	$AP_{Cal.}$ (arcsec/JC)	$AP_{Obs./NEM}$[s] (arcsec/JC)
Mercury	6.0236×10^6	0.38709893[b]	0.20563069[b]	87.969343	87.969257[l]	42.980	42.980 ± 0.001[t]
Venus	4.08523719×10^5	0.72333199[c]	0.00677323[c]	224.70069	224.70079922[m]	8.625	8.6247 ± 0.0005[u]
Earth	3.329460487×10^5	1.00000011[d]	0.01671022[d]	365.25641	365.25636[n]	3.839	3.8387 ± 0.0004[u]
Mars	3.09870359×10^6	1.52366231[d]	0.09341233[d]	686.95999	686.98[o]	1.351	1.3624 ± 0.0005[v]
Jupiter	1.04734864×10^3	5.20336301[d]	0.04839266[d]	4333.2863	4332.589[l]	0.062	0.070 ± 0.004[v]
Saturn	3.4979018×10^3	9.53707032[d]	0.05415060[d]	10756.1991	10755.698[p]	0.014	0.014 ± 0.002[v]
Uranus	2.290298×10^4	19.19126393[d]	0.04716771[d]	30707.48958	30685.4[q]	0.002	
Neptune	1.941226×10^4	30.06896348[d]	0.00858587[d]	60223.35242	60189[r]	0.001	
Pluto	1.3657×10^8	39.4450697[e]	0.250248713[e]	90487.2766	90487.2769[e]	0.000	
Ceres	2.12×10^9	2.76565525349[f]	0.078392019894[f]	1679.946655	1679.94665588[f]	0.304	
Eris	1.19×10^8	68.0038[g]	0.43347[g]	204832	204832[g]	0.000	
Haumea		42.991717[h]	0.198926[h]	102962	102961.7[h]	0.000	
Makemake		45.2801[h]	0.165384[h]	111291	111290.9[h]	0.000	
Gonggong		67.428[i]	0.49795[i]	2.0224×10^5	2.0224×10^5[i]	0.000	
Quaoar		43.5460[h]	0.041365[h]	104959	104959[h]	0.000	
Orcus		39.0977[j]	0.229319[j]	89295	89295	0.000	
Sedna		510.4[i]	0.85036[i]	4.212×10^6	4.21×10^6[i]	0.000	
Pallas	9.71×10^9	2.773814788[g]	0.22975842[g]	1687.3867	1687.386677[g]	0.316	
Vesta	7.41×10^9	2.361659442[f]	0.0883512981[f]	1325.636135	1325.636135[f]	0.451	
Icarus		1.078092896[k]	0.82694358[k]	408.867606	408.867606[k]	10.061	10.1[w]

[a]Observed mass ratio from [32] as approximation of η. [b]Observation data of Epoch J2000, updated on Aug. 19, 2021 [33], uncertainty unspecified. [c]Updated on May17, 2021, *ibid.* [d]Updated on Nov. 25, 2020, *ibid.* [e][34], uncertainty unspecified. [f]Observation data of Jul. 1, 2021, Solution of Apr. 13, 2021 [34]. [g]Solution of Oct. 7, 2021, *ibid.* [h]Solution of Oct. 8, 2021, *ibid.* [i] Solution of Nov. 10, 2021, *ibid.* [j]Solution of Aug. 25, 2021, *ibid.* [k]Solution of Aug. 12, 2021, *ibid.* [l][35], revision Apr. 12, 2021. [m]Revision Oct. 19, 2020, *ibid.* [n]Revision Aug. 15, 2018, *ibid.* Revision Oct. 29, 2019, *ibid.* [p]Revision Jan. 3, 2020, *ibid.* [q]Revision Sep. 30, 2021, *ibid.* [r]Revision May 3, 2021, *ibid.* [s]Result extracted from observation data or by data fitting with Newton-Einstein model (NEM) or combination thereof. [t][36]. [u][37]. [v][38]. [w][39].

[36], and majority of the advancement was interpreted as due to the interactions with other Planets via original LNG.

The issue is that restmass of particle in field is not field invariant by LME but function of field experienced by particle. Therefore, effect of interactions complying with field-dependent restmass of particle shall not be identical to that by original LNG/NEM. Therefore, the question arises that, if against the background of refined LNG, would APM still be ~43″/JC (hence $n_g = -2$)? In concerns of such, the assertion as expressed in Expression (91) is not yet beyond reasonable doubt.

On the other hand, regardless of value of n_g, existence of AP of object in orbital motion in field, by itself, demonstrates that restmass of object in field does increase, whether time is defined on AT or not.

17. Speed of Light in Vacuo

By international agreement, SLV is defined as [41]

$$c \equiv \mathcal{N}_c \mathbb{U}_L \mathbb{U}_{\mathrm{AT}}^{-1} \rightarrow c_x \equiv \mathcal{N}_c \mathbb{U}_{L,x} \mathbb{U}_{\mathrm{AT},x}^{-1} = \mathcal{N}_c \mathbb{U}_L \mathbb{U}_{\mathrm{AT},x}^{-1}. \tag{120}$$

x : State indicator of entity in association.

As has analyzed, duration of unit of AT depends on geometry of space, state of motion, field environment, etc. Therefore, by definition of Expression (120), SLV must also depend on geometry of space, state of motion, field environment, etc. Therefore, in general, SLV defined on AT is state variant instead of SIT.

Stated implicitly in Expression (120) is the invariance of numeral aspect of SLV. In general, by ROM,

$$c_x \equiv n_{c,x} \mathbb{U}_{v,x},\ \mathbb{U}_{v,x} \equiv \mathbb{U}_{L,x} \mathbb{U}_{t,x}^{-1} \rightarrow n_{c,x} = c_x / \mathbb{U}_{v,x} \subset \mathrm{SIT} \rightarrow c_x = n_c \mathbb{U}_{L,x} \mathbb{U}_{t,x}^{-1}. \tag{121}$$

That is, numeral aspect of SLV is SIT regardless of choice/definition of units of length/time. This invariance is often mistaken as state invariance of c and regarded as a principle of physics [42]. State invariance of numeral aspect of SLV was solidified in form of assigned immutable numeral in definition of SLV by international agreement.

By definition, velocity of a particle in motion in space is ratio of length of path the particle travels through and duration of the journey the particle takes during. Photon is particle and vacuum is region of space. Therefore, length of photon path in vacuo is an entity measurable via unit of space, *i.e.*, length of unit of length. Therefore, if locally defined common time of photon path exists then time of the path is definable hence SLV is an entity measurable via length and time units defined thereupon. However, in general, if clocks are not comoving with photon but instead with photon path then locally defined common time of photon path may not exist even if length of the path is infinitesimal. Notwithstanding, n_c shall remain as SIT regardless.

Therefore, the 1983 agreement [41] was not about SLV but definition of unit of length,

$$c_i \equiv \mathcal{N}_c \mathbb{U}_L \mathbb{U}_{\mathrm{AT},i}^{-1} \rightarrow \mathbb{U}_L \equiv c_i \mathbb{U}_{\mathrm{AT},i} / \mathcal{N}_c. \tag{122}$$

$\mathbb{U}_{\mathrm{AT},i}$: Unit of RAT.

That is, unit of SI length, meter, is defined as an assigned fraction of length of photon path in vacuo in RS free of field of any kind during unit duration of RAT. Accordingly, the meter bar (used to be the primary unit of length in SI system) becomes a secondary unit of meter, if distance between designated pair of marks on the metallic prototype is unaltered during transfer of the object from RS to preservation site in France. Accordingly,

$$\frac{c_x}{c_i} = \frac{n_c}{\mathcal{N}_c} \frac{\mathbb{U}_{\mathrm{AT},i}}{\mathbb{U}_{\mathrm{AT},x}} \rightarrow n_c = \mathcal{N}_c \rightarrow c_x = \frac{\mathbb{U}_{\mathrm{AT},i}}{\mathbb{U}_{\mathrm{AT},x}} c_i. \tag{123}$$

Since duration of unit of AT is not SIT therefore SLV is not and cannot be SIT, regardless of theory/assumption.

18. Unit of Rest Time

Duration of unit of time is state variant if time is defined on atomic clock. Motion/field shall cause variation of duration between consecutive clock events generated by the atomic clock therein. For atomic clocks stationed around the globe, such variation, hence ticking rates of the clocks, is location/time dependent and of plurality of periodic features [43]. Not all these periodicities can be eliminated by statistical processing of clock events generated by the clocks. Therefore, it would be better to regard SI second as defined on atomic clock in RS, *i.e.*, at rest in RF free of field of any kind,

$$\begin{matrix} \nu\left[{}^{133}\mathrm{Cs}_{0,\mathrm{hfs},\boldsymbol{i}}\right] \equiv \mathcal{N}_t\ \mathrm{s}^{-1} \\ \mathcal{N}_t \equiv 9\ 192\ 631\ 770 \end{matrix} \rightarrow \mathrm{s}_{\boldsymbol{i}} \equiv \frac{\mathcal{N}_t}{\nu\left[{}^{133}\mathrm{Cs}_{0,\mathrm{hfs},\boldsymbol{i}}\right]} \rightarrow \mathbb{U}_{\mathrm{AT},\boldsymbol{i}} \equiv \mathrm{s}_{\boldsymbol{i}}\,. \tag{124}$$

RAT thus defined is intrinsically uniform by specification of the time. Some of the variations of duration of unit of AT in practice, caused by motion of the clock and/or field experienced, may be detectable/correctable. For instance, with relative precision of atomic fountain clock approaching 1×10^{-16} [44] and that of optical lattice clock to 2×10^{-18} [45], daily variations of AT on Earth may become measurable with array of high precision atomic clocks stationed globally.

By LPE,

$$\mathbb{U}_{\mathrm{AT},\boldsymbol{i}} = \mathcal{N}_t h_{\boldsymbol{i}} / {}^{133}_{\mathrm{hfs},0}\Delta E_{s,\boldsymbol{i}} \rightarrow \mathbb{U}_{\mathrm{AT},x} = \mathcal{N}_t h_x / \Delta E_{s,x}\,. \tag{125}$$

$\mathcal{N}_t$: Immutable numeral assigned by definition of unit of AT. ΔE_s : Selfenergy difference of particle defining unit of AT. $\mathbb{U}_{\mathrm{AT}}$: Unit of AT. x : State indicator of entity in association.

That is, if Planck constant is not SIT, *i.e.*, if h is state dependent, then duration of unit of AT may or may not be function of state of particle defining AT, pending on specific dependency of h on state. Consequently, SLV may or may not be function of state of particle defining AT, pending on specific dependency of h on state. Therefore, validity of **Assumption 1** is of vital importance.

19. Planck Constant

In symbolic form, Planck constant is expressed as

$$h \equiv n_h \mathbb{U}_h,\ \mathbb{U}_h \equiv \mathbb{U}_E \mathbb{U}_t \rightarrow h_x = n_{h,x}\mathbb{U}_{h,x},\ \mathbb{U}_{h,x} = \mathbb{U}_{E,x}\mathbb{U}_{t,x}\,. \tag{126}$$

n_h : Numeral aspect of Planck constant. $\mathbb{U}_h$: Unit of Planck constant. $\mathbb{U}_E$: Unit of energy. $\mathbb{U}_t$: Unit of time. x : State indicator of entity in association.

By ROM,

$$n_{h,x} = h_x / \mathbb{U}_{h,x} \subset \mathrm{SIT} \rightarrow n_{h,x} = n_h \rightarrow h_x = n_h \mathbb{U}_{E,x}\mathbb{U}_{t,x}\,. \tag{127}$$

That is, if expressed in *in-situ* units, numeral aspect of Planck constant, n_h, is SIT regardless of choice/definition of energy/time. Therefore, **Assumption 1** is identical/equivalent/indifferent to the **assumption** that composite entity $\mathbb{U}_E\mathbb{U}_t$ is SIT,

$$h \subset \text{SIT} \;\to\; \mathbb{U}_E \mathbb{U}_t \subset \text{SIT}\,. \tag{128}$$

That is, **Assumption 1** is identical/one and same/indifferent to **assume** composite entity $\mathbb{U}_E \mathbb{U}_t$ is independent of state wherein the entity is measured/defined regardless of choice/definition of energy/time. Since Expression (128) is an **assumption** on the relationship between unit of *energy* and unit of *time*, such must be subjected to experimental falsification/scientific scrutiny.

On the other hand, translation symmetry of infinite space, hence relativity of free motion therein, guarantees the invariance of Planck constant with respect to translational free motion therein, regardless of choice/definition of energy/time [10]. As analyzed in previous sections, LEC in conjunction with LPE guarantees the invariance of Planck constant as measured at rest in SGF, hence centripetal force field in general, regardless of choice/definition of energy/time. Optical experiment on atomic clock in free motion [46] has proven that, up to precision of the experiment, Planck constant defined on AT is invariant to free motion of entity in physical space [10]. GPS clock comparisons also indicated that, to precision of the measurements, Planck constant defined on AT is invariant to free motion of entity in SGF [27]. Therefore, **Assumption 1** is indeed true and valid in such cases, at least to precision of the experiments. That is also why measurements of Planck constant could have reached such high accuracy [47] without much attention to states of the associated entities/units during such measurements. Nevertheless, it is unknown yet if Planck constant would remain as invariant under unconstrained acceleration and/or in other types of force fields. Therefore, Expression (128) shall remain as an **assumption** even though such has always been built in, explicitly or implicitly, in most theories of physics *a prior*.

Under **Assumption 1**,

$$\mathbb{U}_{E,x} \mathbb{U}_{t,x} = \mathbb{U}_{E,\boldsymbol{i}} \mathbb{U}_{t,\boldsymbol{i}} \;\to\; \frac{\mathbb{U}_{E,x}}{\mathbb{U}_{E,\boldsymbol{i}}} \frac{\mathbb{U}_{t,x}}{\mathbb{U}_{t,\boldsymbol{i}}} = 1\,. \tag{129}$$

That is, state function of unit of energy and that of time are of reciprocal relationship regardless of choice/definition of energy/time. Accordingly,

$$\textbf{If}\;\; \mathbb{U}_{t,x} \neq \mathbb{U}_{t,\boldsymbol{i}} \;\;\textbf{then}\;\; \mathbb{U}_{E,x} \neq \mathbb{U}_{E,\boldsymbol{i}};\; \textbf{If}\;\; \mathbb{U}_{E,x} \neq \mathbb{U}_{E,\boldsymbol{i}} \;\;\textbf{then}\;\; \mathbb{U}_{t,x} \neq \mathbb{U}_{t,\boldsymbol{i}}\,. \tag{130}$$

That is, if unit of time is state variant then unit of energy must also be state variant, reciprocally. Likewise, if unit of energy is state variant then unit of time must also be state variant, reciprocally. In other words, if dilation of unit of time exists then inflation of unit of energy must also exist, regardless of choice/definition of energy/time. Likewise, should any process/situation causes alteration of energy of unit of energy then duration of unit of time shall also be altered thereby, reciprocally, regardless of choice/definition of energy/time.

Selfenergy of object is an attribute of same. By definition of attribute, with ROM,

$$E_{s,x} = n_{E,s,x} \mathbb{U}_{E,x} \;\to\; n_{E,s,x} = E_{s,x} / \mathbb{U}_{E,x} \subset \text{SIT} \;\to\; E_{s,x} = n_{E,s} \mathbb{U}_{E,x}\,. \tag{131}$$

$E_{s,x}$: Selfenergy of object in x-state. $n_{E,s,x}$: Numeral aspect of $E_{s,x}$. $\mathbb{U}_{E,x}$: Unit of energy in x-state.

Therefore, with Equation (129),

$$\frac{E_{s,x}}{E_{s,i}} = \frac{\mathbb{U}_{E,x}}{\mathbb{U}_{E,i}} \rightarrow \frac{E_{s,x}}{E_{s,i}} \frac{\mathbb{U}_{t,x}}{\mathbb{U}_{t,i}} = 1. \tag{132}$$

That is, if any process shall alter selfenergy of particle then unit of time of same shall also be altered, reciprocally, regardless of choice/definition of energy/time. Centripetal force field does alter selfenergy of particle at rest therein and **Assumption 1** is valid thereat. Therefore, time dilation in such field is inevitable regardless of choice/definition of energy/time. Therefore, locally defined common time of field does not and cannot exist (except in region of equal potential) whether or not time is defined on AT.

By definition of SLV, Expression (121), with Equation (132),

$$c_x / c_i = \mathbb{U}_{t,i} / \mathbb{U}_{t,x} \rightarrow c_x / c_i = E_{s,x} / E_{s,i}. \tag{133}$$

c_x: SLV as measured/defined in x-state.

That is, if any process alters selfenergy of particle then SLV of same shall also be altered, in same proportion, regardless of choice/definition of energy/time. Centripetal force field does alter selfenergy of particle at rest therein and **Assumption 1** is valid thereat, therefore, SLV in such field is not and cannot be field invariant but function of location in field (except in region of equal potential) whether or not SLV is defined on AT.

Denote

$$\beta_x \equiv c_x / c_i \rightarrow c_x = \beta_x c_i,\ \mathbb{U}_{t,x} / \mathbb{U}_{t,i} = \beta_x^{-1},\ \mathbb{U}_{E,x} / \mathbb{U}_{E,i} = \beta_x^{+1}. \tag{134}$$

β_x: State function.

From Equation (133), by LME,

$$E_{s,x} = \beta_x^{+1} E_{s,i} \rightarrow m_{s,x} = \beta_x^{-1} m_{s,i}. \tag{135}$$

$m_{s,x}$: Selfmass of object in x-state.

That is, if any process shall reduce selfenergy of a particle then selfmass of the particle shall be increased reciprocally, regardless of choice/definition of energy/time. Centripetal force field does reduce selfenergy of particle at rest therein and **Assumption 1** is valid thereat, therefore, selfmass increase of particle in such field is inevitable, whether or not time is defined on AT.

From Expression (125), with Expression (127),

$$\mathbb{U}_{\mathrm{AT},x} = \frac{\mathcal{N}_t n_h \mathbb{U}_{E,x}}{\Delta E_{s,x}} \mathbb{U}_{\mathrm{AT},x} \rightarrow \frac{\Delta E_{s,x}}{\mathbb{U}_{E,x}} = \mathcal{N}_t n_h \subset \mathrm{SIT} \rightarrow n_h = \frac{1}{\mathcal{N}_t} \frac{\Delta E_{s,x}}{\mathbb{U}_{E,x}}. \tag{136}$$

Therefore, if selfenergy difference of particle defining AT could be calculated in terms of energy of unit of energy in same state then Planck constant would become calculable. Alternatively, if $\Delta E_{s,x}$ could be measured with respect to unit of energy in same state then Planck constant would become measurable without involvement of time.

Not long ago, the international community has adopted a new definition for

Planck constant [18]. Accordingly, h is now defined as

$$h \equiv 6.626\ 070\ 15 \times 10^{-34}\,\mathrm{J\ s} \;\rightarrow\; h \equiv \mathcal{N}_h \mathbb{U}_{E,\mathrm{SI}} \mathbb{U}_{t,\mathrm{SI}}\,. \tag{137}$$

$\mathcal{N}_h$: Immutable numeral assigned by new definition of Planck constant.

Accordingly, from Equation (136),

$$n_h = \mathcal{N}_h \;\rightarrow\; \mathbb{U}_{E,x} = \frac{\Delta E_{s,x}}{\mathcal{N}_h \mathcal{N}_t} \equiv \mathrm{J}_x \;\rightarrow\; \mathbb{U}_{E,i} = \frac{\Delta E_{s,i}}{\mathcal{N}_h \mathcal{N}_t} \equiv \mathrm{J}_i\,. \tag{138}$$

Therefore, the new definition of h is not really about Plank constant *per se* but new definition of unit of *energy*. That is, SI unit of energy, Joule, is now redefined *de facto* as assigned plurality/fraction of selfenergy difference of particle defining SI time, *i.e.*, AT. In comparison with the abandoned unit of energy, the new definition of unit of energy is superior in that (a) raising unit of *energy* to primary status; (b) explicit connection of unit of *energy* with that of *time*; (c) avoiding notion of *force*. Drawback of such system of units is that energy, mass, time, SLV, and attributes derived from/depended on such entities directly and/or indirectly shall become state sensitive. Therefore, related LOPs and associated constants/parameters may become more sophisticated than otherwise, which, on the other hand, is inevitable due to LME and **Assumption 1**.

By definition, with definition of Expression (138), (122), and LME,

$$\mathbb{U}_E \equiv \frac{\mathbb{U}_m \mathbb{U}_L^2}{\mathbb{U}_t^2} \;\rightarrow\; \mathbb{U}_{m,i} = \frac{\mathbb{U}_{E,i} \mathbb{U}_{\mathrm{AT},i}^2}{\mathbb{U}_{L,i}^2} = \frac{\mathcal{N}_c^2}{\mathcal{N}_h \mathcal{N}_t} \frac{\Delta E_{s,i}}{c_i^2} = \frac{\mathcal{N}_c^2}{\mathcal{N}_h \mathcal{N}_t} \Delta m_{s,i} \equiv \mathrm{kg}_i\,. \tag{139}$$

$\mathbb{U}_m$: Unit of mass, kg in SI system.

That is, SI unit of mass, kg, is now redefined *de facto* as assigned plurality/fraction of selfmass difference of particle defining SI time. Accordingly, the kilogram prototype (used to be primary unit of *mass* in SI system) becomes a secondary unit of mass upon correction with Schwarzschild Factor and Lorentz Factor at preservation site of the kilogram account for effect of state of motion/field environment of the site.

There is also a consequence of Expression (128) that, by ROM,

$$\frac{\Delta E_x}{\mathbb{U}_{E,x}} \subset \mathrm{SIT},\ \frac{\Delta t_x}{\mathbb{U}_{t,x}} \subset \mathrm{SIT} \;\rightarrow\; \frac{\Delta E_x \Delta t_x}{\mathbb{U}_{E,x} \mathbb{U}_{t,x}} \subset \mathrm{SIT} \;\rightarrow\; \frac{\Delta E_x \Delta t_x}{\Delta E_i \Delta t_i} = 1\,. \tag{140}$$

That is, composite of alteration of energy and time is SIT during any process regardless of choice/definition of energy/time. Therefore, any such aspect, hence those derived/derivable therefrom, of any process can be described in RF on RT if dependency of states of the entities involved therein is addressable/addressed.

20. Refined Law of Newton Gravitation in Finite Space

Consider a mass particle in RS in $\mathbf{S}^3$ (three-dimensional finite space) with its location assigned as internal origin of RF, for convenience. Then, there shall exist SGF in association with the particle in the space with center of the field at origin. Such field can be probed by test particle (effect of probe particle to the field being probed is ignored/disregarded by definition of *test* particle). According to refined LNG, Equation (63), gravitation force probed by probe is

$$\boldsymbol{F}_{p,x} = -G_{f,x}^{1/2} G_{p,x}^{1/2} m_{f,x} m_{p,x} f_s \boldsymbol{e}_\varphi, \ x = f, 0, g. \tag{141}$$

$\boldsymbol{F}_{p,x}$: Gravitation force probed by probe at rest in field of particle. $G_{f,x}$: Gravitation constant as measured at rest at location of particle. $G_{p,x}$: Gravitation constant as measured at rest at location of probe. $m_{f,x}$: Restmass of particle as measured at rest at location of same. $m_{p,x}$: Restmass of probe as measured at rest at location of same. *s*: Internal distance between particle and probe. f_s : Function of *s*. $\boldsymbol{e}_\varphi$: Unit vector of ***s*** at location of probe.

By definition of gravitation field,

$$\boldsymbol{G} \equiv \frac{\boldsymbol{F}_p}{m_p} \to \boldsymbol{G}_{f,x} = \frac{\boldsymbol{F}_{p,x}}{m_{p,x}} = -G_{f,x}^{1/2} G_{p,x}^{1/2} m_{f,x} f_s \boldsymbol{e}_\varphi. \tag{142}$$

$\boldsymbol{G}$: Gravitation field. m_p : Restmass of probe. $\boldsymbol{G}_{f,x}$: SGF associated with particle.

In classical field theories, field of an object is an intrinsic property of the object alone that is unaltered regardless of presence/absence and/or action/inaction of others. Accordingly, plurality of fields in space is vector additive. However, due to LME and state invariance of Planck constant in field, presence of mass in field of mass particle shall cause alteration of the restmass and associated gravitation parameter of the particle therefore alteration of the field of the mass particle. Therefore, classical field is incompatible with LME, and SGF as expressed in Expression (142) is dependent on others, hence fields are not simple vector additive but interactive.

By flux conservation theorem of Gauss for gravity, *i.e.*, the law of Gauss gravitation,

$$-\oiint_S \frac{\boldsymbol{G} \cdot d\boldsymbol{S}}{4\pi G} = m \to -\oiint_{\text{SPS}} \frac{\boldsymbol{G}_{f,x} \cdot d\boldsymbol{S}}{4\pi G_{f,x}^{1/2} G_{p,x}^{1/2}} = m_{f,x}. \tag{143}$$

S: Any nonlocal simultaneous enclosure of simple connectivity. $d\boldsymbol{S}$: Areal element of the enclosure. *G*: Gravitation constant. *m*: Mass enclosed by the enclosure. SPS: Spherical probing sphere, *i.e.*, enclosure of radius *s* concentric to field.

With spherical coordinate system [10],

$$d\boldsymbol{S}_\varphi = \boldsymbol{e}_\varphi R_e^2 \sin^2\varphi \sin\vartheta \, d\vartheta \, d\theta \to f_s = R_e^{-2} \sin^{-2}\left[s/R_e\right]. \tag{144}$$

$d\boldsymbol{S}_\varphi$: Areal element of SPS. R_e : External radius of $\mathbf{S}^3$.

Therefore, LNG in $\mathbf{S}^3$ is expressed as, for field centered at origin,

$$\boldsymbol{F}_p = -\frac{G_i m_s^* m_p^*}{R_e^2 \sin^2\left[s/R_e\right]} \boldsymbol{e}_\varphi. \tag{145}$$

$\boldsymbol{F}_p$: Gravitation force experienced by particle in field. G_i : Gravitation constant as measured in RS. m_s^* : Restmass parameter of particle causing field. m_p^* : Restmass parameter of particle at rest in field. *s*: Internal distance between particle and center of field. $\boldsymbol{e}_\varphi$: Unit vector of ***s*** at location of particle in field.

In vicinity of origin,

$$s \ll R_e \to \sin\frac{s}{R_e} \simeq \frac{s}{R_e} \to \boldsymbol{F}_g \simeq -\frac{G_i m_s^* m_g^*}{s^2} \hat{\boldsymbol{s}}. \tag{146}$$

That is, in near field of origin (in comparison to R_e), LNG is of the familiar form, as approximation.

From Equation (15) and Expression (91),

$$\frac{d\beta_g^4}{d\rho} = \frac{4}{R_g^2 \sin^2\left[\rho / R_g\right]} \rightarrow \beta_g = \left(1 - \frac{4}{R_g}\cot\frac{\rho}{R_g}\right)^{1/4}, \quad \begin{matrix} 0 \le \beta_g \le 1 \\ \rho \le \pi R_g / 2 \end{matrix}. \tag{147}$$

β_g: Schwarzschild Factor for particle in SGF in $\mathbf{S}^3$. R_g: External radius of $\mathbf{S}^3$ in unit of CLF. ρ: Internal distance of particle to origin, in unit of CLF.

For finite space, no point outside SS is free of field of any kind except SF. Therefore, reference state for field is chosen at internal radius of finite space, whereat, strength of field centered at internal origin is minimal hence an approximation to true RS.

21. Retardation of Self Field of Gravitation

Consider a mass particle in geodesic motion with respect to RF. Then, SF of the particle, *i.e.*, gravitation field associated with the particle, shall be of ellipsoidal symmetry with respect to the geodesic, since velocity of field propagation is finite. According to Gauss gravitation law, Equation (143), therefore, there must exist mass at trail of the particle in motion, referred to herein as phantom mass, which is caused by deviation of SF of particle in motion from spherical symmetry. Although phantom in naming, such mass is as real and physical as that of the particle in motion according to Gauss gravitation law. Further, phantom mass of particle in motion is attractive to mass of same since it is of same type as that of the particle in motion. Consequently, motion of mass particle in finite space shall be retarded by phantom mass induced by SF of particle in motion. Therefore, SF of mass particle is dissipative in energy correlating to velocity of absolute motion of mass particle in finite space.

According to refined LNG, under ISA,

$$\frac{d\boldsymbol{P}_{f,u,g}}{dt_{s,u,g}} = \boldsymbol{F}_{f,u,g} = -\lambda' \frac{G_i m'_{f,0,g} m_{f,0,g}}{\beta_g'^2 \beta_g^2 r_g^2} \frac{\boldsymbol{v}_{f,u,g}}{c_i}, \ r_g \equiv \frac{G_i m_i}{c_i^2}. \tag{148}$$

$\boldsymbol{P}_{f,u,g}$: Momentum of particle in motion in SF as perceived at rest at particle location in SF. $t_{s,u,g}$: ST of particle in motion in SF. $\boldsymbol{F}_{f,u,g}$: Gravitation force experienced by particle in motion in SF. λ': Positive scalar parameter. $m'_{f,0,g}$: Phantom restmass of particle in motion in SF. $m_{f,0,g}$: Restmass of particle in motion in SF. β'_g: Schwarzschild Factor of phantom mass in SF. β_g: Schwarzschild Factor of particle in motion in SF. $\boldsymbol{v}_{f,u,g}$: Velocity of particle in absolute motion in RF as perceived at rest at particle location in SF. r_g: CLF of SF of particle. G_i: Gravitation constant as measured in RS free of any field. m_i: Restmass of particle in RS. c_i: SLV as measured/defined in RS.

In general, phantom mass induced by nonsphericity of SF of a particle in motion is not particle-like but of extent and profile pending on velocity of absolute motion of the particle. Deviations of particle model to such aspects are absorbed in the working parameter above. Internal distance between particle in motion and phantom mass at trail thereof is also not r_g but monotonically relates to r_g and velocity dependent as well. Such relationship and dependency are also absorbed by the working parameter in addition to Schwarzschild Factors and other considerations/approximations. Thus, equation of motion of the particle can be expressed as

$$\beta_u \frac{d}{d\tau}\left(\frac{\boldsymbol{u}}{\beta_u}\right) = -\lambda \boldsymbol{u},\ \ \beta_u \equiv \sqrt{1-u^2},\ \ d\tau \equiv \frac{c_i}{r_g} dt_i. \tag{149}$$

$\boldsymbol{u}$: Velocity of particle in absolute motion, in reduced unit. τ: RT rescaled with CLF of SF and SLV in RS. λ: Parameter function. t_i : RT.

The working parameter λ is a function of velocity of particle in absolute motion, among other things, and regarded herein as a constant, as zero-order approximation of the subject. Then,

$$\frac{1}{1-u^2}\frac{du^2}{d\tau} = -2\lambda u^2 \ \rightarrow \ u = \frac{u_0}{\sqrt{u_0^2 + \left(1-u_0^2\right)e^{2\lambda\tau}}},\ u_0 \equiv u\big|_{\tau=0}. \tag{150}$$

That is, without cause of others, mass particle tends to be at rest with respect to RF unless it is in motion at SLV. Therefore, if gravitation is factored in and propagation velocity of field finite, there shall be no such thing as inertial motion of mass object in finite space even if the object were to along geodesic in absence of external interference of any kind. In addition, kinetic energy of mass particle in motion in finite space shall be dissipated via interaction of the particle with SF and, by LEC, such energy shall not vanish from finite space.

In comparison, charge particle in geodesic motion in finite space shall also cause deviation of SF of the charge from spherical symmetry, due to finite velocity of field propagation. According to the flux conservation theorem, such deviation shall cause phantom charge to appear at trail of charge in motion. Although phantom in naming, phantom charge of charge in motion is as real and physical as charge of charge particle by the theorem. Further, phantom charge is of same sign as that of charge in motion therefore repulsive to charge in motion. Consequently, motion of charge in finite space would have been accelerated by phantom charge induced by SF of charge in motion and that would have caused LEC violation. However, according to Maxwell electrodynamics [20], absolute motion of charge shall create transient magnetic field in association with motion of the charge and the transient field shall further create transient electric field hence electric force against motion of the charge. Therefore, no net effect of phantom charge nor violation of LEC shall be observed, other than the existence of electromagnetic field in association with charge in absolute motion in finite space.

22. Discussion

Under the law of $E_p = h\nu$ and the law of energy conservation, Planck constant is and must be invariant in centripetal force field. Therefore, phenomenon of time dilation shall occur in centripetal force field since such field alters selfenergy of mass object at rest therein. As a consequence, c and G in field are not and cannot be field invariant but state variant by definition of the attributes, due to invariance of Planck constant thereunder, regardless of choice/definition of energy/time. Nevertheless, field invariance of *c* and *G* were assumed *a prior* in most gravitation theories including GRT while time dilation of clock in field was first predicted by GRT, indicating internal inconsistency in GRT. Theory/interpretation aside, relativistic gravitation phenomena are shown derivable/derived, without internal inconsistency, from refined LNG compatible with $E = mc^2$, LEC, and other common laws of physics. It is also suggested to reevaluate Mercury's anomalous precession of 43″/JC under the new paradigm or seek for alternatives to determine/verify the parameter n_g in refined LNG.

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Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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