



Prognostics and Health Monitoring Methodologies and Approaches: A Review

Hasan A. Bjaili^{1*} and Ali M. Rushdi¹

¹*Department of Electrical and Computer Engineering, King Abdulaziz University, P.O. Box 80204
Jeddah 21589, Saudi Arabia.*

Authors contributions

This work was carried out in collaboration between both authors. Both authors read and approved the final manuscript.

Article Information

DOI: 10.9734/JERR/2020/v18i417215

Editor(s):

(1) Dr. Harekrushna Sutar, Indira Gandhi Institute of Technology, India.

Reviewers:

(1) Bashar Jabbar Hamza, Al-Furat Al-Awsat Technical University, Iraq.

(2) Fidias E. Leon-Sarmiento, University of Pennsylvania, Colombia.

Complete Peer review History: <http://www.sdiarticle4.com/review-history/62205>

Received 27 August 2020

Accepted 03 November 2020

Published 18 November 2020

Short Research Article

ABSTRACT

Prognostics is a term that engineering borrowed from medicine to refer to the discipline concerned with the Remaining Useful Life (RUL) of an engineering device. This paper surveys the RUL prediction techniques and classifies them into four categories of model-based techniques, knowledge-based techniques, experience-based techniques, and data-driven techniques. A comparative review is given for the main features, prominent advantages, potential shortcomings and main subcategories for each of these categories. The survey is supported by an extensive list for up-to-date references.

Keywords: Prognostics; State-space modeling; algorithms; Remaining Useful Life (RUL); lithiumion battery; reliability.

2010 Mathematics Subject Classification: 53C25, 83C05, 57N16.

**Corresponding author: E-mail: hasan@bjaili.com;*

1 INTRODUCTION

Prognostics is a term introduced recently into the reliability engineering discipline. This term (borrowed from that of medical prognostics) is concerned with the Remaining Useful Life (RUL) [1] (also known as the Mean Residual Life (MRL)) of engineering devices. It focuses on predicting the time in which the device will no longer perform its intended function. It is the field of predicting the future reliability and performance of an engineering device by assessing the deviation or degradation extent of the device from its normal operation condition expectation [2].

In recent years, the research on prognostics, reliability and asset life prediction has been enhanced in the field of Engineering Asset Management (EAM). Modern systems such as aircraft engines are often built with enormous complexities. These systems are often bundled with rich electronics and intricate connections as well as interactions among their subsystems and components. For example, a typical car consists of about 30,000 parts, 10 million lines of software code and 2,000 functional components [3]. An engine manufacturer such as Rolls-Royce® has developed a service agreement called TotalCare™ in which engines are not sold, but instead they are leased with a TotalCare™ service agreement. The cost of lease under the TotalCare™ service agreement is actually paid by the hour. The lease requires a support fee per hour of operation and the Original Equipment Manufacturer (OEM) or Engine Manufacturer provides the service support, including tracking of the inherent uncertainty. This service is also called *Power-By-The-Hour*. The Power-By-The-Hour policy eliminates the requirement for the lessee to pay hourly *maintenance reserves* for the use of the engines in addition to a fixed monthly lease fee. In the Power-By-The-Hour service, the lessee is protected from the unexpected cost of premature engine failure or capital cost to refurbish the supplier's owned engines that require replacement due to life-limited parts (LLPs), Airworthiness Directive (AD) compliance and performance degradation. The lessor is responsible for the cost to refurbish or repair engines that require removal from the aircraft and a shop visit to resolve discrepancies.

1.1 Life-Limited Parts (LLPs)

Life-limited Parts (LLPs) are parts that have a limited service life. When this service life has been exhausted, the life-limited part must be removed from the aircraft and replaced before the aircraft is permitted to fly again. The service life of the life-limited parts may be expressed in hours of operation, cycles of operation, or calendar time.

1.2 Airworthiness Directives (AD)

Airworthiness Directives (AD) are legally enforceable regulations issued by the Saudi Arabian's General Authority for Civil Aviation (GACA), United States Federal Aviation Administration (FAA), European Aviation Safety Agency (EASA) or any other country's civil aviation authorities. These regulations are used to correct an unsafe condition in a product which could be an aircraft, engine, propeller, or appliance.

The Power-By-The-Hour service has led to a shape change in the product Life Cycle Cost (LCC) profile for the Original Equipment Manufacturer (OEM). In order for the engine manufacturer or lessor to account and budget for unexpected failures and LCC and still make it appealing for the lessee or operator, the OEM had to invest in research concerning the Mean Residual Life (MRL), which is frequently referred to as the Remaining Useful Life (RUL). Therefore, scientists studied and applied many life prediction approaches and techniques. Failure of an engineering device can be traced to an implicit deterioration and degradation mechanism that acts and evolves over time. Thus, the understanding and identification of the different potential failure mechanisms present in engineering devices is essential for accurate RUL and life prediction [4, 5, 6, 7]. The RUL and life prediction techniques can be classified into four categories (see Fig. 1):

- Model-Based Techniques,
- Knowledge-Based Techniques,
- Experience-Based Techniques,
- Data-driven Techniques.

Each of the following four sections is devoted to an overview of one the aforementioned categories.

2 MODEL-BASED TECHNIQUES

The term "Model-based techniques" typically refers to approaches using models derived from first principles (e.g., physics-based). It simply depicts methods that use mathematical models of system behavior. In essence, the model can be derived from either one of the following two ways:

- knowledge (when available) from first principles, known physical laws, expert experience, dimensional analysis, method of similitude, prototyping, and the like.
- when a large amount of data is available for both nominal and degraded behavior.

Models are typically developed from a mixture of system knowledge and system data. Systems can be described by discrete state-space equations of the form:

$$\begin{aligned} \mathbf{x}(k+1) &= \mathbf{f}(k, \mathbf{x}(k), \phi(k), \mathbf{u}(k), \mathbf{v}(k)), \\ \mathbf{y}(k) &= \mathbf{h}(k, \mathbf{x}(k), \phi(k), \mathbf{u}(k), \mathbf{n}(k)), \end{aligned} \quad (2.1)$$

where \mathbf{x} : states, ϕ : parameters, \mathbf{u} : input, \mathbf{y} : output, \mathbf{v} : process noise, \mathbf{n} : sensor noise.

A model-based technique is used when an accurate and explicit mathematical model of the degradation process can be developed and constructed from first principles [8, 5]. It utilizes the outcome of a consistency check between the sensed measurement of the actual system and the outcomes of a mathematical model. The difference is called the residual. If the residual is large, then there is a malfunction, and when the residual is small, then there is a normal disturbance, noise or modeling error [9]. The model-based approach is also called Analytical-Based or Accelerated Degradation Modeling (ADM).

The pros of the model-based technique include the features that it:

- relies on the understanding of the physics of the system,
- experiences changes in the feature vector due to and commensurate with a change in the model parameters [10],
- requires less data than the data-driven approach,

- demands lower cost for implementation.

On the other hand, the cons of the model-based technique are manifested in the fact that it

- requires more knowledge on the fundamental theory relevant to the monitored system,
- demands many assumptions about the practical operating conditions of the system,
- suffers from the dependence of its robustness and accuracy on the experimental condition under which the models were developed [11],
- requires the estimation of various physical parameters of the system,
- might not produce desirable and practical results since the fault type differs from one component to another.

The model-based technique can be further subcategorized as [12]:

- Physics-of-Failure (PoF) Models,
- Statistical Models,
- Kalman/Particle Filtering, or
- Non-linear Dynamics.

2.1 Physics-of-Failure (PoF) Models

The physics-of-failure (POF) provides a focus for the life and reliability aspects of components. It addresses the root causes of failure such as fatigue, fracture, wear, and corrosion [13]. The technique of Physics-of-Failure (PoF) prognostics depends on the product life cycle loading and failure mechanism to get the Remaining Useful Life (RUL) and perform prognostics evaluation [14]. This technique allows assessing the product reliability under its actual usage conditions. It utilizes the product model as well as *in situ* measuring sensor data integrated together to assess the product deviation and degradation from its expected normal operation or operating conditions and predict of the future reliability state. Fig. 2. outlines the Physics-of-failure (PoF) based prognostics and health monitoring (PHM) methodology. First, the design data, life cycle, expected life cycle conditions, Failure Modes, Mechanisms and Effects Analysis (FMMEA) [14, 15, 2, 16, 17, 18], and Physics-of-failure models

are combined to be the inputs and are called the virtual life assessment (Virtual Reliability). The critical failure modes and mechanisms are prioritized based on the virtual life assessment. The built-in-test results, existing sensory data, inspection and maintenance records, and warranty data are also utilized to identify the abnormal conditions. The monitoring parameters and sensor location can be determined using the aforementioned information. It is impossible to perform data-driven prognostics for a product that has not been manufactured since there is no data available for training the algorithm. For new-product prognostics, we only need to change the material geometries or properties to model the product. This is because most new products are not completely different from previous products. Similar products can be referenced via Failure Modes, Mechanisms and Effects Analysis (FMMEA) [14]. FMMEA is a systematic approach to identify failure mechanisms and models for all potential failure modes and then prioritize them. It is based on understanding the relationships between [19]:

- Requirements and the physical

characteristics of the product.

- The interactions of product materials with loads.
- The influence on product failure susceptibility with respect to the use conditions.

The methodology of PoF-based prognostics comes in handy when prognostics analysis is needed for a new product or for the prognosis of a legacy system. In the case of legacy systems, it is difficult to obtain training data and also very hard to assess the Remaining Useful Life (RUL) if the failure mechanisms and their effect on the collected parameters are not well understood. The PoF-based prognostics approach depends on the understanding of the legacy system structure and life cycle conditions as well as the failure modes and mechanisms.

According to [20], the PoF technique can be summarized as:

- Establish a list of the probable fault structure for example chemical, electrical, physical, mechanical, structural, or thermal processes leading to failure.

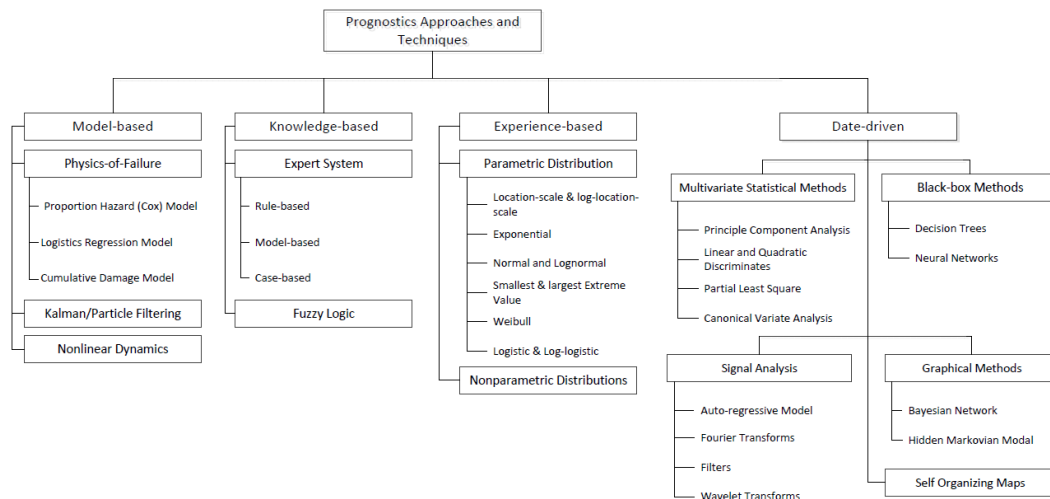


Fig. 1. Taxonomy of prognostics approaches and techniques

2.2 Statistical Models

Statistical modeling (which is also called the probabilistic-based modeling or usage-based prognostics modeling) is typically developed based on known information about the degradation of the measure parameters [21]. The parameters measurements are collected over time to assess the current severity of the parameters distribution shifts. Changes in these parameters can be accelerated using the accelerated degradation techniques.

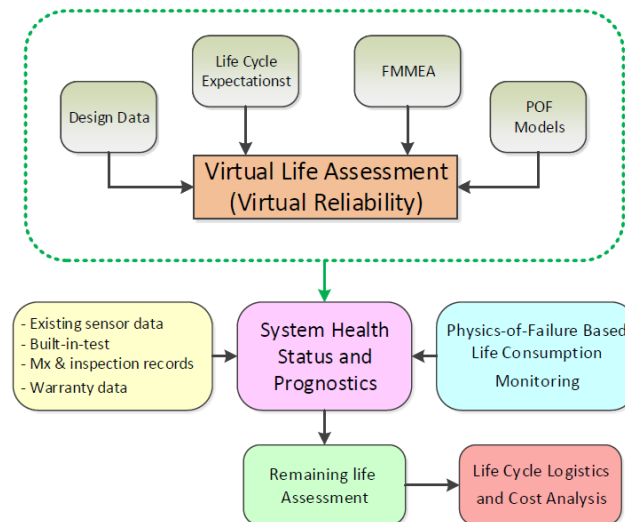


Fig. 2. PoF-based PHM methodology. Graph reproduced from [14]

Accelerated Life Test (Accelerated Degradation)

The accelerated life tests consist of different test techniques to shorten the life of products or to hasten the degradation of their performance. The purpose of such testing is to obtain degradation data quickly. Such data is then modeled and analyzed. We do this because in real life, devices of practical utility take long time to deteriorate or degrade. The real life degradation process is usually very slow at normal wear-and-tear conditions, *i.e.*, the Mean Time To Failure (MTTF) is comparatively high. Therefore, to obtain statistical data in a fast manner from a degradation test, we employ an accelerated life test by changing and speeding up the environment variables such as temperature, vibration amplitude, load, voltage, corrosive media and pressure [22, 23]. There are many types of acceleration testing such as:

- **High Usage Rate:** Also called compressed time testing, is an easy

method to hasten the deterioration and shorten the life of many products by running them at a higher usage rate. We can achieve such testing by either running the component **faster** or by **reducing its time off** period. An example of running a component faster is when a rolling bearing is run at three or four times its intended or normal speed. **Reduced time off** accelerated testing concerns home appliances such as washers and dryers when they are run 24-hour a day while in normal life they are run one or two hours a day. Another example of reduced time off accelerated testing is when a toaster or a coffee maker are cycled hundreds of times a day while in normal life they are cycled three to five times a day.

- **Overstress Testing:** In overstress accelerated testing, the product runs in an overstressed environment to shorten

its life. This is done by changing the component overstressed environment variables such as temperature, thermal cycling, humidity, vibration, voltage and/or mechanical load.

- Stress Loading** Stress loading is a type of accelerated testing methods that is done by applying a stress on the component until it fails while degradation statistical data is being gathered. Stress loading can be carried out using constant stress, step stress, or linearly increasing stress. The constant stress refers to the case in which constant stress or force is applied while the component is running. The step stress can be done by applying stress to the component while changing the stress setting at specified times. The stress is applied at low level and at a specified time. If the component does not fail, the stress is raised and held for a determined time and so on and so forth until the component is failed. Simple step stress uses only two stress levels. The method of linearly increasing stress means simply applying stress and increasing it linearly until the component fails.

There are many statistical (probabilistic-based) models. However, this paper will only review three of them, which are:

2.2.1 Proportional Hazards Model (Cox Model)

The proportional hazards model (also called Cox model) is a type of a regression method [24]. This model is widely used in biomedical applications to investigate the effects of several covariates on a survival distribution at the same time. Cox [25, 26, 27] who introduced this model assumes no particular type of distribution for the survival data. Because Cox model is non-parametric (distribution-free), it cannot extrapolate in time but in stress. Cox model is a statistical technique for exploring the relationship between the survival of the subject (component, part, human being, etc.) in test and several explanatory variables. The survival analysis is mainly concerned with studying the time between entry to the study and a subsequent event such as inoperability (cease of operation), breakage, damage, etc. [27].

Cox proportional hazard model or Cox model cater for estimating the hazard (or risk) of parts/equipment damage, given its prognostics variables. The model is briefly explained below:

Let x_1, \dots, x_n denote the variables, and let $h_0(t)$ denote the unknown life distribution hazard function at $x_1 = x_2 = \dots = x_n = 0$. Thus, the hazard function for the distribution at the variables x_1, \dots, x_n is:

$$h(t; x_1, \dots, x_n) = h_0(t) \cdot e^{(\gamma_1 x_1 + \dots + \gamma_n x_n)}. \quad (2.2)$$

The base hazard function $h_0(t)$ and the coefficients $\gamma_1, \gamma_2, \dots, \gamma_n$ are estimated from data. The corresponding reliability function is:

$$R(t; x_1, \dots, x_n) = e^{[-\int_0^t h(\tau; x_1, x_2, \dots, x_n) d\tau]} = [R_0(t)]^{e^{(\gamma_1 x_1 + \dots + \gamma_n x_n)}}, \quad (2.3)$$

where $R_0(t)$ is the reliability function at $x_1 = x_2 = \dots = x_n = 0$ and in given by

$$R_0(t, 0, 0, \dots, 0) = e^{[-\int_0^t h_0(\tau) d\tau]}. \quad (2.4)$$

The Proportional Hazards Model (Cox Model) is presented in detail in [26, 28, 29].

2.2.2 Logistic Regression Model

The logistic regression model is also widely used in biomedical applications in which the predictor or dependent variable is a binary 0 or 1, *on* or *off*, *operational* or *inoperational*, *dead* or *alive*, *approved* or *disapproved*, etc. This model could also be called the qualitative response/discrete choice model. The logistic regression model is a probability distribution of an event occurring depending on the values of the independent variables, which can be categorical or numerical. The model estimates the probability that an event occurs for a randomly selected observation versus the probability that the event does not occur. It seeks the effect of a series of variables on binary response variables and classifies observations by estimating the probability that an event observation is a particular category, as mentioned above, *on* or *off*, etc.

The logistic proportional model for the proportion p in a particular category, for example, *inoperational* as a function of n independent variables x_1, \dots, x_n is

$$\ln \left[\frac{1-p}{p} \right] = \gamma_0 + \gamma_1 x_1 + \dots + \gamma_n x_n, \quad (2.5)$$

where $\gamma_0, \gamma_1, \dots, \gamma_n$ are unknown coefficients to be estimated from the data.

2.2.3 Cumulative Damage Model

The **Cumulative Damage Model** is defined to be the permanent (irreversible) damage accumulation in a component under a cyclical usage pattern [30]. It is developed based on the concepts of bounds on residual fatigue life in two-stage cycling [31]. Basically, it has been used in three different areas [32]:

- **Mechanical Systems:** to predict time of failure
- **Health and Safety:** to determine the humans tolerance level when a person is exposed to toxic or latently injurious materials.
- **Structural Engineering:** to calculate the structure safety of structural parts when they are exposed to loads over time.

The cumulative damage can be modeled using the non-linear Miner's rule [6]:

$$D = \left(\frac{n}{c}\right)^r \quad (2.6)$$

Where D is the damage, n is the number of cycles experienced, r is the non-linear damage exponent, and c is the number of cycles to crack initiation.

2.3 Kalman/Particle Filtering

Filtering is usually used in many situations in engineering. For example, radio communication signals are usually corrupted with noise which makes the signal unusable. Therefore, a good filtering algorithm may come in handy to remove noise [33]. In this subsection, we will discuss only two types of filtering that could be used in prognostics to estimate the model parameters: *Kalman Filters* and *Particle Filters*. Filtering using Kalman/Particle filters has been used in [34, 35] to predict the state of a battery charge and the Remaining Useful Life (RUL).

2.3.1 Kalman Filtering

The Kalman filter is a statistical estimator to the linear-quadratic problem, which is technically the problem of estimating the instantaneous *state* of a linear dynamic system affected by a white

noise [36]. Alternatively, Kalman filtering can be viewed as a recursive algorithm which can estimate the true (instantaneous) state of a noisy system [37]. Kalman filtering can only be applied to linear systems. Non-linear system can be filtered using the *Extended Kalman Filter* which is an improved version of the original Kalman filter. A linear system is simply a process that can be described by the following state (difference) and output equations:

$$\begin{aligned} \vec{x}_{k+1} &= \vec{A}\vec{x}_k + \vec{B}\vec{u}_k + \vec{w}_k \\ \vec{y}_k &= \vec{C}\vec{x}_k + \vec{z}_k \end{aligned} \quad (2.7)$$

where \vec{A} , \vec{B} , and \vec{C} are matrices, \vec{u} is a known control input, \vec{y} is the measured output, \vec{z} is the measured noise, \vec{w} is the process noise, and k is the discrete time index.

The \vec{x} , \vec{u} , \vec{w} , \vec{y} and \vec{z} quantities are vectors, in general; therefore, they each of them might than one element. Kalman filters can expressed in many forms and formulations, one of them is [33]:

$$K_k = AP_k C^T (CP_k C^T + S_z)^{-1}. \quad (2.8)$$

$$\hat{x}_{k+1} = (A\hat{x}_k + Bu_k) + K_k(y_{k+1} - C\hat{x}_k). \quad (2.9)$$

$$P_{k+1} = AP_k A^T + S_w - AP_k C^T S_z^{-1} CP_k A^T. \quad (2.10)$$

2.3.2 Particle Filtering

Particle filters are sequential Monte Carlo statistical simulation techniques which can provide a very accurate approximation of the sequential Bayesian estimator [38]. The basic concept of the particle filters as in [39] is described next. We use a set of weighted particles as follows:

$$\left\{ w_{k-1}^{(i)}, \mathbf{x}_{k-1}^{(i)} \right\}_{i=1}^N \quad (2.11)$$

where $\mathbf{x}_{k-1}^{(i)}$ is the state of the particle i while the weight of this particle is $w_{k-1}^{(i)}$. These are used to approximate the posterior density at a time $k-1$. The weights are normalized, i.e., $\sum_{i=1}^N w_{k-1}^{(i)} = 1$. This approximation, then, can be expressed as:

$$p(\mathbf{x}_{k-1} | \mathbf{z}_{1:k-1}) \approx \sum_{i=1}^N w_{k-1}^{(i)} \delta_{\mathbf{x}_{k-1}^{(i)}}(\mathbf{x}_{k-1}) \quad (2.12)$$

Where $\delta_a(\mathbf{x})$ is a Dirac delta that is concentrated at \mathbf{a} and $\mathbf{z}_{1:k-1}$ is available measurements up to time $k-1$. Prognostics using Particle filtering and particle filters are described in detail in [40, 41, 34, 42, 43].

2.4 Nonlinear Dynamics

Linear dynamical systems can be described as objects of any type or nature. The state of a dynamical system evolves over time in accordance with dynamical rules or as a result of an evolving deterministic operator [44]. Every dynamical system has a mathematical model associated with it, A dynamical system is considered as a black box whose internal parts are unknown as shown in Fig. 3.

In this simple method, it is assumed that $\mathbf{y}(t)$, a time series quantity, is obtained by observing the underlying autonomous dynamical system and is mixed with unmonitored and unwanted force or noise [45]. Inherently, there is another time series quantity which we also observe, that is the input $\mathbf{u}(t)$.

There are many approaches to develop a model for the dynamical system. The deterministic approach, one of them, assumes that the input-output time series relationships can be modeled

as:

$$\frac{d\mathbf{x}}{dt} = f(\mathbf{x}(t), \mathbf{u}(t)), \quad (2.13)$$

$$\mathbf{y}(t) = h(\mathbf{x}(t)). \quad (2.14)$$

Many researchers used the nonlinear dynamics techniques to model components for prognostics analysis. Gašperin *et al.* [46] modeled a gear to predict its Remaining Useful Life (RUL). The propagation of the damage of the gear is inherently a stochastic process; therefore, its distribution of the Remaining Useful Life (RUL) must be estimated by the propagation of the distribution of the current system state. The gear damage was modeled by a nonlinear dynamical system with two hidden states and a single measured output. The gear condition was described with a dynamical process, which is affected by random tribological inputs. These inputs occur due to the impact of the rotating parts. The aforementioned condition can then be described by the following state-space model which is a random process:

$$\mathbf{x}_{t+1} = f(\mathbf{x}_t, \mathbf{w}_t, \Theta) \quad (2.15)$$

$$\mathbf{y}_t = g(\mathbf{x}_t, \mathbf{e}_t, \Theta) \quad (2.16)$$

Prognostics using nonlinear dynamical systems are described in detail in [47, 48].

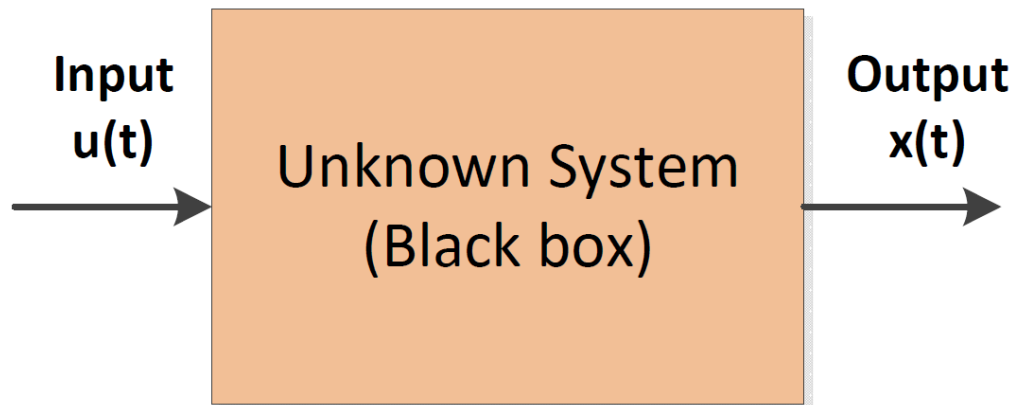


Fig. 3. Conceptual model of a single-input, single output system

3 KNOWLEDGE-BASED TECHNIQUES

When it is difficult to perform prognostics analysis on engineering devices using model-based techniques, the knowledge-based techniques may come in handy. Knowledge-based prognostic techniques predict the Remaining Useful Life (RUL) by assessing the similarity between a data bank of a predefined failures and an observed current situation [11]. The predefined failure data are usually collected from subject-matter experts as well as via an interpretation of a set of rules [49]. Fuzzy logic and expert systems are the most common techniques used to predict the Remaining Useful Life (RUL).

3.1 Expert Systems

An expert system is mainly a computer program or a set of computer programs that emulates the performance and decision-making ability of a knowledgeable human being. A knowledge-based expert system consists of either one of the following:

- Rule-based expert system,
- Model-based expert system,
- Case-based expert system.

3.1.1 Rule-based Expert System

In a rule-based expert system, the knowledge is represented in an IF **Condition(s)** THEN **action(s)** such as, "If Exhaust Gas Temperature (EGT) is more than 650° THEN the inlet fans are malfunctioned." These **IF-THEN** statements form a knowledge database comprised of heuristic factual data gathered by many experts over many years as shown in Fig. 4. In order for any rule-based expert system to be useful, it ought to be complete as well as exact as much as possible [50]. For a *complete* rule-based expert system, all rules and information available should have been one-to-one mapped. To be *exact*, the aforementioned mapping should be logically consistent such that no chain of logical rules pulled from the knowledge base contradicts with the human operator. A rule-based system is often called a **production system** [51].

3.1.2 Model-based Expert System

A model-based expert system is used on a good model that mimics the structure and behavior of an engineering device. Generally, a model-based expert system, *not to be confused with Model Based Techniques previously discussed in section 2*, requires deeper reasoning to predict the Remaining Useful Life (RUL). In a model-based expert system, there are three fundamental tasks for prognostics:

1. Hypothesis Generation,
2. Hypothesis Testing,
3. Hypothesis Discrimination.

It is assumed that there is only a single point of failure [52]. The explanation of the aforementioned tasks is described in detail in [52]. In this process, the source of the problem prediction and the use of the observation get separated to improve the confidence of each problem source. The modeled system's behavior is then compared with the observed behavior of the same system and if they differ from one other, then there is a discrepancy in the system provided the model is correct and accurate.

3.1.3 Case-based Expert System

In our normal life, we tend to look for the services of experienced/older doctors and professionals. This is because we think that the experienced doctors or professionals have been around for a while and treated/fixes similar issues in the past; therefore, they quickly can figure out the remedy. A case-based expert system is a technique that depends on past experience (cases) to find a solution for the existing/current problem. Similarly, to the experienced doctors or professionals, a case based method takes information on a current behavior of a problem and searches for the most similar past issue/problem (case). One of the advantages and strengths of the case-based expert systems is that they become better and better overtime when more cases are added and analyzed. The knowledge-based expert systems are all explained in detail in the Artificial Intelligence (AI) Literature in [51, 53, 54, 55, 56, 57, 52].

3.2 Fuzzy Logic

Fuzzy logic was presented in 1965 by Zadeh [58]. Basically, it was an extension to the classical Boolean logic in order to relax the tough constraint of the Excluded Middle (of being either absolutely *true* or absolutely *false*). Zadeh suggested that why not have something like 0.75 *true* [59]? The aforementioned 0.75 *true* statement can be interpreted as "**not really true**". Fuzzy logic theory alleviates the harsh constraint of the classical logic set theory of *true* and *false* by allowing partial set association based on a

variable degree of truth. It provides a mechanism for the linguistic phrases such as *low*, *medium*, *high*, *often*, *few*, *etc.* in a nonlinear mapping of the vector of the input data into a scalar single output [60].

A fuzzy prognostics system comprises of: a knowledge base, a fuzzy rule database and a program (algorithm) as shown in Fig. 5. The algorithm fuzzifies the crisp input, utilizing both databases to make inferences based on the fuzzified input, produces a fuzzy output, and finally defuzzifies it into a crisp output.

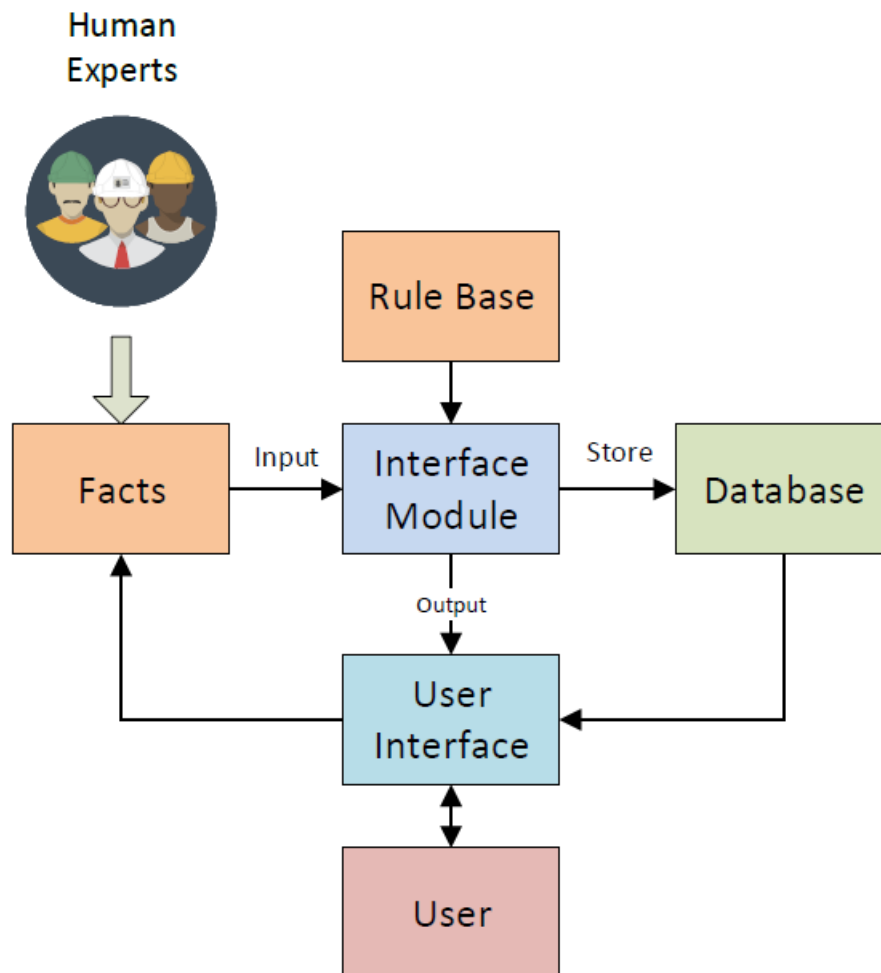


Fig. 4. Rule-based architecture

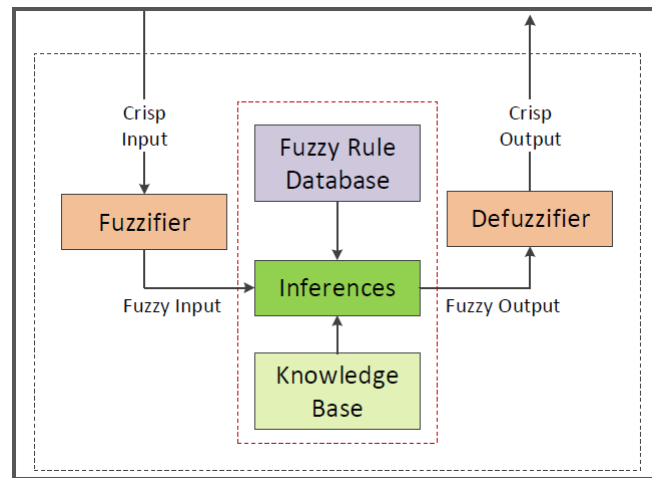


Fig. 5. Fuzzy logic system

Similarly to all knowledge-based systems, the fuzzy logic system uses the **IF-THEN** rules to solve a problem, but unlike the other knowledge-based systems, fuzzy logic is intentionally made imprecise [11]. Fuzzy logic systems are regularly used to enhance other prognostics models [61, 62, 63, 64]. Majidian and Saidi [65] predicted the Remaining Useful Life (RUL) of a set of boiler tubes using fuzzy logic algorithms.

With its robust mathematical framework, fuzzy logic can deal with real-life imprecision and non-statistical uncertainty. Fuzzy logic and its applications are discussed in abundance in [66, 59, 67, 68, 69, 58, 65, 61, 62, 63, 64].

4 EXPERIENCE-BASED TECHNIQUES

Probably the simplest technique of prognostics is based on the well-known probability distributions. Those are distribution functions based on identical events records of failures logged over a period of time. The experience-based technique is extensively used in reliability analysis. When studying probability, it is assumed that some probabilities and/or related quantities can be calculated based on this knowledge assumption. Conversely, in mathematical statistics, data is observed to compute related quantities and/or other probabilities or to predict the RUL [70].

The method of experience-based models depends mainly on mathematical statistics which is classified as *parametric* or *non-parametric*. This classification depends on our knowledge of the data distribution that belongs to a given family in which parameters such as the mean (μ) and the standard deviation (σ) can easily be computed.

4.1 Parametric Distributions

A parametric distribution can concisely be described with just a few parameters without having to report the entire curve. A parametric model can be used to extrapolate—in time—to either the upper tail of a distribution or the lower part. It provides smooth estimates of the failure-time distribution which are perfect for prognostics analysis.

The following parametric distributions can be used for prognostics to predict the RUL:

- Location-scale and Log location-scale,
- Exponential,
- Normal and Lognormal,
- Smallest and Largest Extreme Value,
- Weibull,
- Logistic and Log-Logistic.

4.1.1 Location-Scale & Log Location-Scale Distribution

The Location-Scale is one of the parametrized probability distribution families. It is parametrized by a *location* and a *scale*. For any random variable X where x belongs to the location-scale family, its CDF is a function of $(x - a)/b$. The Location-Scale CDF is:

$$F_x(X|a, b) = F\left(\frac{x - a}{b}\right) \quad -\infty < a < \infty, b > 0 \quad (4.1)$$

If the distribution of X is absolutely continuous the (a, b) is the location-scale parameter with a pdf:

$$f_x(x|a, b) = \frac{1}{b} f\left(\frac{x - a}{b}\right) \quad (4.2)$$

Transforming the location-scale distribution using $X = \log(Y)$ in which $Y = e^x$ produces a log-location-scale distribution. Based on this transformation, the location parameter μ becomes $\theta = e^\mu$ and the scale parameter σ becomes $\lambda = 1/\sigma$ where θ is called the scale parameter and λ is called the power parameter [71].

Mukhopadhyay and Roy [72] used the log-location-scale distribution to perform Bayesian accelerated life testing as well as reliability analysis. The model used is:

$$f_{Y_j} = (y|\theta_j, \tau_j, \mathbf{z}) = \tau_j f_j(\tau_j(y - \mu_j(\theta_j, \mathbf{z}))) \quad (4.3)$$

4.1.2 Exponential Distribution

The exponential distribution which is also called the *negative exponential distribution* is a probability distribution that describes a time between two events occurring. The pdf of the exponential distribution is:

$$f(x) = \begin{cases} \lambda e^{-\lambda x}, & \text{if } x \geq 0 \\ 0, & \text{if } x < 0 \end{cases} \quad (4.4)$$

The exponential distribution as a one-parameter distribution can be used in prognostics by taking the hazard function to be constant, $\lambda(t) = \lambda > 0$, all over the range of T [26]. The failure frequency is:

$$f(t) = \lambda e^{-\lambda t} \quad (4.5)$$

Hence, the CDF is

$$F(t) = 1 - e^{-\lambda t} \quad (4.6)$$

Therefore, the reliability function will be

$$R(t) = e^{-\lambda t} \quad (4.7)$$

The reliability function which is also called the survival function [73] has a mean (μ) and variance (σ^2) as follows:

$$\mu = \frac{1}{\lambda} \quad (4.8)$$

$$\sigma^2 = \frac{1}{\lambda^2} \quad (4.9)$$

Gebraeel *et al.* [74] modeled the exponential degradation function as:

$$S(t_i) = \phi + \theta e^{(\beta t_i)} e^{\left(\epsilon(t_i) - \frac{\sigma^2}{2}\right)} \quad (4.10)$$

where $S(t_i)$ denotes the degradation signal as a continuous stochastic process, continuous with respect to time t , $t_i \geq 0$, $i = 1, 2, \dots$, ϕ is a constant deterministic parameter, θ is a lognormal random variable and $\ln(\theta)$ has a mean μ_0 and variance σ_0^2 , β is a normal random variable with a mean μ_1 and variance σ_1^2 , $\epsilon(t_i)$ is a random error part following a normal distribution with a zero mean and variance σ^2 .

The hazard rate will then be:

$$h(t) = \lambda \quad (4.11)$$

which is constant and independent of time. The only distribution that has a constant failure rate is the exponential distribution [75].

Based on that, the Mean Time Between Failures (MTBF) is $1/\lambda$ and the failure rate is λ while $1/\lambda$ is the 63rd percentile meaning that 63% of the population will have failed by then.

Failure experts claim that electronic components fail randomly, thus, exhibiting exponential distribution failure. The exponential distribution is no longer used heavily in reliability applications [76].

4.1.3 Normal and Log-Normal Distribution

Without any doubts, the most famous and most widely used probability distribution model is the *Normal Distribution* which is also known as the

Gaussian Distribution. A random variable X with pdf:

$$f(x; \mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \quad (4.12)$$

is a normal random variable with the mean μ ($-\infty < \mu < \infty$) and standard deviation $\sigma > 0$.

The normal distribution hazard function increases without limits. Therefore, it can be used to model products with wear-out failure such as the life of incandescent lamp filaments as well as the electrical insulation. Nelson [27] used the normal distribution Cumulative Distribution Function (CDF) to model the population fraction failing by age y .

$$F(y) = \int_{-\infty}^y \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx, \quad -\infty < y < \infty \quad (4.13)$$

The mean μ and standard deviation σ should be in the same measurement of y whatever it is, hours, months or cycles.

Evaluating 4.13 at $\mu = 0$ and $\sigma = 1$ yields:

$$F(y) = \Phi \left[\frac{(y - \mu)}{\sigma} \right], \quad -\infty < y < \infty \quad (4.14)$$

Which is the standard normal distribution function. Additionally, Daniel Inman *et al.* [77] used the normal distribution to model the statistical strength and toughness of composite materials under dispersed type load sharing as:

$$G_n(\sigma) = \Phi \left(\frac{\sigma - \mu_n}{\gamma_n} \right) \quad \sigma \geq 0 \quad (4.15)$$

where

$$\Phi(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^z e^{-\frac{t^2}{2}} dt \quad (4.16)$$

with mean μ and standard deviation γ_n .

The **Lognormal distribution** is widely used to predict metal fatigue, solid state components and electrical insulation life. Engineering devices that follow the lognormal distribution can be modeled as:

$$F(t) = \Phi \left[\frac{(\log(t) - \mu)}{\sigma} \right], \quad t > 0 \quad (4.17)$$

which is the population fraction by age t . Note that $\log(\cdot)$ here is base e usually written as $(\ln(\cdot))$ and should not be confused with base 10. Nelson [27] showed the lognormal reliability function to be:

$$R(t) = 1 - \Phi \left[\frac{(\log(t) - \mu)}{\sigma} \right] \quad (4.18)$$

4.1.4 Smallest and Largest Extreme Value Distribution

The smallest and largest extreme value distribution is characterized by *scale* and *location* parameters. The smallest extreme value distribution is skewed to the left and is used to model the minimum value from a distribution of random observations. It is also used to model time to failure for a system that fails when its weakest component fails. Whereas, the largest extreme value distribution is skewed to the right and is used to model the maximum value from a distribution of random observations. It is also used to describe extreme phenomena such as extreme velocities.

The smallest extreme value distribution is used, also, to predict the life of a series system in which the systems fails if any component fails. Whereas, the largest extreme value is used to predict the life of a parallel system in which the system fails when all its components fails.

The probability density function (pdf) of the smallest extreme (the minimum) is:

$$f(x) = \frac{1}{b} e^{\left(\frac{x-a}{b}\right)} e^{-e^{\left(\frac{x-a}{b}\right)}}, \quad -\infty < x < \infty \quad (4.19)$$

where a is the location and b is the scale parameter.

The pdf of the largest extreme (the maximum) is [78, 75]:

$$f(x) = \frac{1}{b} e^{\left(-\frac{x-a}{b}\right)} e^{-e^{\left(-\frac{x-a}{b}\right)}}, \quad -\infty \leq x \leq \infty \quad (4.20)$$

where again a is the location and b is the scale parameter. Note that the two exponentials in equation 4.20 have extra negative sign when compared with the corresponding ones in equation 4.19.

4.1.5 Weibull Distribution

The Weibull distribution is widely used in reliability analysis. It is frequently used to model the time to failure. The pdf of the Weibull distribution is:

$$f(x) = \frac{\beta}{\delta} \left(\frac{x}{\delta}\right)^{\beta-1} e^{-\left(\frac{x}{\delta}\right)^\beta}, \quad \text{for } x > 0 \quad (4.21)$$

where δ is the scale parameter and is > 0 and β is the shape parameter and is also > 0 .

The pdf of a Weibull distribution as used in prognostics analysis is:

$$f(t) = \frac{\beta}{\eta} \left(\frac{t-\gamma}{\eta} \right)^{\beta-1} e^{-\left(\frac{t-\gamma}{\eta}\right)^\beta} \quad (4.22)$$

where η is the characteristic life, β is the shape factor and the γ is the location parameter. Thus the Hazard rate of a Weibull distribution is:

$$h(t) = \frac{\beta}{\eta} \left(\frac{t-\gamma}{\eta} \right)^{\beta-1}, \quad t \geq \gamma; \eta > 0 \quad (4.23)$$

The Cumulative Distribution Function (CDF) of a Weibull distribution is:

$$F(t) = 1 - e^{-\left(\frac{t-\gamma}{\eta}\right)^\beta}, \quad t \geq \gamma; \beta, \eta > 0 \quad (4.24)$$

The reliability function can be expressed as follows:

$$R(t) = 1 - F(t) = e^{-\left(\frac{t-\gamma}{\eta}\right)^\beta}, \quad t \geq \gamma; \beta, \eta > 0 \quad (4.25)$$

4.1.6 Logistic and Log-logistic Distribution

The logistics distribution is usually used for growth models. Its Probability Density Function (pdf) is:

$$F(t) = \frac{e^{\frac{(t-\mu)}{\sigma}}}{\sigma \left(1 + e^{\frac{(t-\mu)}{\sigma}} \right)^2}, \quad (4.26)$$

$$-\infty < t < \infty, -\infty < \mu < \infty, \sigma > 0$$

where μ is the location parameter and σ is the scale parameter. The logistic failure rate and the reliability functions are:

$$\lambda(t) = \frac{e^{\frac{(t-\mu)}{\sigma}}}{\sigma \left(1 + e^{\frac{(t-\mu)}{\sigma}} \right)^2} \quad (4.27)$$

$$R(t) = \frac{1}{1 + e^{\frac{(t-\mu)}{\sigma}}} \quad (4.28)$$

The Probability Density Function (pdf) of the log-logistic distribution is:

$$f(t) = \frac{\left(\frac{\beta}{\alpha}\right) \left(\frac{t}{\alpha}\right)^{\beta-1}}{\left[1 + \left(\frac{t}{\alpha}\right)^\beta\right]^2}, \quad \alpha > 0, \beta > 0 \quad (4.29)$$

The survivor and hazard functions are shown below:

$$S(t) = \frac{1}{1 + \left(\frac{t}{\alpha}\right)^\beta} \quad (4.30)$$

$$h(t) = \frac{\left(\frac{\beta}{\alpha}\right) \left(\frac{t}{\alpha}\right)^{\beta-1}}{1 + \left(\frac{t}{\alpha}\right)^\beta} \quad (4.31)$$

The log-logistics distribution got its name due to the fact that it handles a random variable Y given by:

$$Y = \log T \quad (4.32)$$

where T is the logistic random variable. The pdf of Y is given by:

$$f_Y(y) = \frac{(b^{-1}) \left(e^{\frac{y-\mu}{b}} \right)}{\left(1 + e^{\frac{y-\mu}{b}} \right)^2} \quad (4.33)$$

where $\mu = \log \alpha, b = \beta^{-1}, -\infty < \mu < \infty, b > 0$.

4.2 Non-parametric Distributions

When it is difficult to fit the data to a known or specific probability distribution family, a non-parametric distribution is used. A method using this type of distribution is also called a *distribution-free* method or technique [75]. Additionally, no assumption about the distribution of the underlying population is made other than that it is continuous [79].

5 DATA-DRIVEN APPROACHES

Data driven approaches depend on large sets of time series data. They use statistical or artificial intelligence techniques on relatively large sets of measured component degradation or performance data to predict component reliability and states. Many of the existing approaches of the data-driven prognostics analysis use neural networks to model the system and hence its RUL. Data driven methods can be grouped into four main categories: multivariate statistical, black-box, signal analysis and graphical methods.

5.1 Multivariate Statistical Methods

Multivariate statistical analysis focuses on analyzing several related variables

simultaneously. Each of those variables is equally important. The most important statistical methods are:

5.1.1 Principal Component Analysis

The technique of principal component analysis is one of the simplest of the multivariate methods. The main objective of this analysis is to take a bunch of variables such as $x_1, x_2, x_3, \dots, x_p$, and then find a combination of these to generate indices $Z_1, Z_2, Z_3, \dots, Z_p$ which are uncorrelated [55]. This lack of correlation indicates that they measure different dimensions of the data and in the order such that $Var(Z_1) \geq Var(Z_2) \geq Var(Z_3) \geq \dots \geq Var(Z_p)$ where $Var(\cdot)$ denotes the variance. Principal component analysis works as follows:

Assume we have a p number of X variables as $x_1, x_2, x_3, \dots, x_p$, then their combination is:

$$Z_1 = a_{11}x_1 + a_{12}x_2 + \dots + a_{1p}x_p \quad (5.1)$$

Then the second principal component:

$$Z_2 = a_{21}x_1 + a_{22}x_2 + \dots + a_{2p}x_p \quad (5.2)$$

The above is chosen so that $Var(Z_2)$ is as large as possible subject to the constraint that:

$$a_{21}^2 + a_{22}^2 + \dots + a_{2p}^2 = 1 \quad (5.3)$$

Since Z_1 and Z_2 have zero correlation for the data. Therefore, the i^{th} principal component is:

$$Z_i = a_{i1}x_1 + a_{i2}x_2 + \dots + a_{ip}x_p \quad (5.4)$$

In particular, $Var(Z_i) = \lambda_i$ and the constants $a_{i1}, a_{i2}, \dots, a_{ip}$, are the elements of the corresponding eigenvectors scaled so that:

$$a_{i1}^2 + a_{i2}^2 + \dots + a_{ip}^2 = 1 \quad (5.5)$$

Future information on the principal components analysis is can be found in [80, 78]

5.1.2 Linear and Quadratics Discriminant Analysis

Discriminant Analysis (DA) is a statistical model which uses a set of independent variables to discriminate between two or more groups of a dependent variable [81]. The linear discriminant analysis (LDA) is, also known as Fisher's linear discriminant, a method used in machine learning,

statistics and pattern recognition. LDA is most commonly used as dimensionality reduction technique to find a linear combination of features that characterizes or separates two or more classes of objects or events. The Quadratic Discriminant Analysis (QDA) is a generalization of the LDA.

The LDA and QDA are the simplest discrimination analyses due to their linearity. A linear decision boundary is easy to understand and visualize. Assume that we draw a sample from a multivariate normal distribution $N(\mu, \Sigma)$ that has a mean μ and covariance matrix Σ . The multivariate density can then be:

$$P(x) = \frac{1}{\sqrt{(2\pi)^D |\Sigma|}} e^{-\frac{1}{2}(x-\mu)^T \Sigma^{-1} (x-\mu)} \quad (5.6)$$

A sample of data is drawn from two classes, each described by a multivariate normal density

$$P(x) = \frac{1}{\sqrt{(2\pi)^D |\Sigma|}} e^{-\frac{1}{2}(x-\mu_k)^T \Sigma^{-1} (x-\mu_k)} \quad (5.7)$$

LDA can be applied as follow: Assume that each of $f_k(x)$, $k = 1, \dots, K$ follows a multivariate normal distribution (μ_k, Σ) with a mean μ_k and common covariance matrix Σ [24]. Then:

$$\begin{aligned} & \log \left[\frac{P(G=k|X=x)}{P(G=j|X=x)} \right] \\ &= \log \frac{f_k(x)}{f_j(x)} + \log \left(\frac{\pi_k}{\pi_j} \right) \\ &= \log[\pi_k/\pi_j - \frac{1}{2}(\mu_k + \mu_j)^T \Sigma^{-1} (\mu_k - \mu_j) \\ & \quad + x^T \Sigma^{-1} (\mu_k - \mu_j)] \end{aligned} \quad (5.8)$$

If 5.8 is greater than zero then k will be grouped in a vector; otherwise j will be grouped instead. For classes $k = 1, 2$ the posterior probability $P(k|x)$ of observing an instance of class k at point x can be found using Bayes rule as follows:

$$P(k|x) = \frac{\pi_k P(k|x)}{p(x)} \quad (5.9)$$

Note that the unconditional probability $P(x)$ does not depend on k . Taking the natural logarithm of the posterior odds:

$$\begin{aligned} \log \frac{P(k=1|x)}{P(k=2|x)} &= \log \frac{\pi_1}{\pi_2} - \frac{1}{2} \log \frac{|\Sigma_1|}{|\Sigma_2|} \\ & \quad + x^T (\Sigma_1^{-1} \mu_1 - \Sigma_2^{-1} \mu_2) \\ & \quad - \frac{1}{2} x^T (\Sigma_1^{-1} - \Sigma_2^{-1}) x \\ & \quad - \frac{1}{2} (\mu_1^T \Sigma_1^{-1} \mu_1 - \mu_2^T \Sigma_2^{-1} \mu_2) \end{aligned} \quad (5.10)$$

We can obtain the hyperplane that separates the two classes by equating this log-ratio to zero [82]. The above is the quadratic function of x . Hence, it is called the *Quadratic Discriminant Analysis* (QDA). If the covariance matrix $\Sigma_1 = \Sigma_2$, then the quadratic terms disappears making it *Linear Discriminant Analysis* (LDA).

5.1.3 Partial Least Squares

Partial Least Squares (PLS) is a regression approach that splits the predictors or independent variables to a smaller set of uncorrelated components then performs least square regression on them rather on the whole original data [83, 84].

The core functionality of PLS is dimension reduction which works under the assumption of a basis latent decomposition of the predictor matrix in which $X \in \mathbb{R}^{n \times p}$ and response matrix in which $Y \in \mathbb{R}^{n \times q}$ as follows:

$$X = TP^T + E \quad (5.11)$$

$$Y = UQ^T + F \quad (5.12)$$

where, $X \in \mathbb{R}^{n \times m}$ is a matrix that is capable of producing k linear combinations known as *scores*, $Y \in \mathbb{R}^{n \times p}$ matrix of responses, $T \in \mathbb{R}^{n \times l}$ is called X-scores $U \in \mathbb{R}^{n \times l}$ is called the Y-scores, $P \in \mathbb{R}^{m \times l}$ and $Q \in \mathbb{R}^{p \times l}$ are matrices of coefficients known as *loadings* and $E \in \mathbb{R}^{n \times p}$ and $F \in \mathbb{R}^{n \times q}$ are random errors matrices.

5.1.4 Canonical Variate Analysis

Canonical Variate Analysis (CVA) is a technique that focuses on finding the relationship between groups of variables in a data set. Assumes that we have two data sets, namely X and Y such that $X = \{x_1, x_2, x_3 \dots x_n\}$ and $Y = \{y_1, y_2, y_3 \dots y_n\}$. The CVA main function is to find whether X and Y are related, *i.e.*, can Y be represented by X ? Basically, CVA finds the linear combinations of X and Y that are highly correlated.

6 CONCLUSION

The concept of the Remaining Useful Life (RUL) studied herein is also known as the Mean Residual Life (MRL) for an engineering device. This is the corner stone of the emerging field of engineering prognostics, which mimics the

older field of medical prognostics, by allowing an engineering device to play a role similar to that of the human body. The Remaining Useful Life is simply a scientific prediction of the time in which an engineering device will no longer perform its intended function. Prognostics is, therefore, the field of predicting the future reliability and performance of an engineering device by assessing the deviation or degradation extent of the device from its normal operation condition expectation.

This paper surveys the prediction techniques for the Remaining Useful Life (RUL) of an engineering device under continuous operation. The paper classifies these techniques into four categories, namely model-based techniques, knowledge-based techniques, experience-based techniques, and data-driven techniques. A comparative exposition is given for the main features, prominent advantages, potential shortcomings and main subcategories for each of these technique categories. The survey is supported by an extensive list for up-to-date references. The present survey is intended to supplement many excellent reviews that address the prognostics of devices of contemporary vital importance [85, 86, 87, 88, 89, 90, 91, 92, 93, 94].

ACKNOWLEDGMENT

This article was funded by the Deanship of Scientific Research (DSR), King Abdulaziz University, Jeddah. The authors, therefore, acknowledge with thanks DSR technical and financial support.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

REFERENCES

- [1] Bjaili Hasan A, Moinuddin, Muhammad Rushdi, Ali M. A state-space backpropagation algorithm for nonlinear estimation. *Circuits, Systems, and Signal Processing*, Springer. 2019;38(8):36823696.

- [2] Pecht Michael. Prognostics and health management of electronics. Wiley Online Library; 2008
- [3] Tsui Kwok L, Chen Nan, Zhou Qiang, Hai Yizhen, Wang Wenbin. Prognostics and health management: A review on data driven approaches. Mathematical Problems in Engineering. Hindawi Publishing Corporation. 2015;2015.
- [4] Gorjian Nima, Ma Lin, Mittinty Murthy, Yarlagadda Prasad, Sun Yong. A review on degradation models in reliability analysis. Engineering Asset Lifecycle Management. Springer. 2010;369384.
- [5] Schwabacher Mark. A survey of data-driven prognostics. Proceedings of the AIAA Infotech@ Aerospace Conference. 2005;1-5.
- [6] Byington Carl S, Roemer Michael J. Prognostic enhancements to diagnostic systems for improved condition-based maintenance [military aircraft]. Aerospace Conference Proceedings, IEEE. 2002;2002(6):6-2815.
- [7] Dragomir Otilia Elena, Gouriveau Rafael, Dragomir Florin, Minca Eugénia, Zerhouni Nouredine. Review of prognostic problem in condition-based maintenance. Control Conference (ECC), 2009 European. IEEE. 2009;1587-1592.
- [8] Baraldi Piero, Cadini Francesco, Mangili Francesca, Zio Enrico. Model-based and data-driven prognostics under different available information. Probabilistic Engineering Mechanics. Elsevier. 2013;32:66-79.
- [9] Luo Jianhui, Namburu Madhavi, Pattipati Krishna, Qiao Liu, Kawamoto Masayuki, Chigusa Shunsuke. Model-based prognostic techniques [maintenance applications]. AUTOTESTCON 2003. IEEE Systems Readiness Technology Conference. Proceedings. IEEE. 2003;330340.
- [10] A dynamical systems approach to damage evolution tracking, part 1: description and experimental application. Chelidze David, Cusumano Joseph P, Chatterjee Anindya. Journal of Vibration and Acoustics. American Society of Mechanical Engineers. 2002;124(2):250257.
- [11] Sikorska JZ, Hodkiewicz Melinda, Ma Lin. Prognostic modelling options for remaining useful life estimation by industry. Mechanical Systems and Signal Processing. Elsevier. 2011;25(5):18031836.
- [12] Fernandes Pedro, Roy Rajkumar, Mehnen Jörn, Harrison Andrew. An overview on degradation modelling for service cost estimation. Functional Thinking for Value Creation. Springer. 2011;309314.
- [13] Sadiku Matthew, Shadare Adebowale, Dada Emmanuel, Musa Sarhan. Physics Of Failure: An Introduction. International Journal of Scientific Engineering and Applied Science. 2016;09(2):108-111.
- [14] Pecht Michael, Gu, Jie. Physics-of-failure-based prognostics for electronic products. Transactions of the Institute of Measurement and Control. SAGE Publications. 2009;31(3-4):309-322.
- [15] A profust reliability based approach to prognostics and health management. Zhao Zhiyao, Quan Quan, Cai Kai-Yuan. Reliability, IEEE Transactions on, IEEE. 2014;63(1):26-41.
- [16] Zhenhua Wen, Yuan Peng Liu. Applications of Prognostics and Health Management in aviation industry. Prognostics and System Health Management Conference (PHM-Shenzhen). IEEE. 2011;2011:15.
- [17] Pei Chun, Fu Guicui, Li Nan, Zou Hang. A research on the application of Prognostics and Health Management to process system. Prognostics and System Health Management (PHM), 2012 IEEE Conference on, IEEE. 2012;15.
- [18] Rushdi Ali Muhammad Ali, Bjaili Hasan Ahmed. An ROBDD algorithm for the reliability of double-threshold systems. British Journal of Mathematics & Computer Science. 2016;19(6):117.
- [19] Pecht Michael. Product reliability, maintainability, and supportability hand. CRC Press; 2009.
- [20] White Mark. Microelectronics reliability: physics-of-failure based modeling and

- lifetime evaluation. Pasadena, CA: Jet Propulsion Laboratory, National Aeronautics and Space; 2008.
- [21] George Vachtsevanos, Frank Lewis, Michael Roemer, Andrew Hess, Biqing Wu. Intelligent fault diagnosis and prognosis for engineering systems. John Wiley & Sons, Inc; 2006.
- [22] Shiau Jyh-Jeu Horng, Lin Hsin-Hua. Analyzing accelerated degradation data by nonparametric regression. IEEE Transactions on Reliability. IEEE. 1999;48(2):149-158.
- [23] Pham Hoang. Reliability modeling, analysis and optimization. World Scientific; 2006.
- [24] Pham Hoang. Springer hand of engineering statistics. Springer Science & Business Media; 2006.
- [25] Lawless JF. Statistical models and methods for lifetime data. Wiley Series in Probability and Statistics, Wiley; 2002. (ISBN 9780471372158, LCCN 81011446)
- [26] Kalbfleisch John D, Prentice, Ross L. The statistical analysis of failure time data. John Wiley & Sons. 2011;360.
- [27] Nelson Wayne B. Accelerated testing: statistical models, test plans, and data analysis. John Wiley & Sons. 2009;344.
- [28] Lee Elisa T, Wang John. Statistical methods for survival data analysis. John Wiley & Sons. 2003;476.
- [29] Hosmer DW, Lemeshow S, May S. Applied survival analysis: regression modeling of time to event data. Wiley Series in Probability and Statistics. Wiley; 2011. (ISBN 9780470258002)
- [30] Usynin Alexander, Hines J, Urmanov Aleksey. Formulation of prognostics requirements. Aerospace Conference, 2007 IEEE. 2007;1-8.
- [31] Ben-Amoz M. A cumulative damage theory for fatigue life prediction. Engineering Fracture Mechanics. Elsevier. 1990;37(2):341-347.
- [32] Saunders Sam C. Cumulative damage models. Encyclopedia of Statistical Sciences. Wiley Online Library; 1982
- [33] Simon Dan. Kalman filtering. Embedded systems programming. 2001;14(6):72-79.
- [34] Saha Bhaskar, Goebel Kai. Modeling Li-ion battery capacity depletion in a particle filtering framework. Proceedings of the annual conference of the prognostics and health management society, 2009;2909-2924.
- [35] Prognostics methods for battery health monitoring using a Bayesian framework. Saha, Bhaskar and Goebel, Kai and Poll, Scott and Christophersen, Jon. Instrumentation and Measurement, IEEE Transactions on, IEEE. 2009;58(2):291-296.
- [36] Mobinder S. Grewal, Angus P. Andrews. Kalman filtering theory and practice using matlab. New York: John Wiley & Sons; 2001.
- [37] Prognostics health management of electronic systems under mechanical shock and vibration using Kalman filter models and metrics. Lall, Pradeep and Lowe, Ryan and Goebel, Kai. IEEE Transactions On Industrial Electronics. IEEE. 2012;59(11):4301-4314.
- [38] A tutorial on particle filters for online nonlinear/non-Gaussian Bayesian tracking. Arulampalam M. Sanjeev, Maskell Simon, Gordon Neil, Clapp Tim. IEEE Transactions on signal processing. IEEE. 2002;50(2):174-188.
- [39] Ristic Branko. Particle filters for random set models. Springer. 2013;798.
- [40] Zio Enrico. Monte Carlo simulation methods for reliability estimation and failure prognostics. Complex Data Modeling and Computationally Intensive Statistical Methods. Springer. 2010;151-164.
- [41] Prognostics 101: A tutorial for particle filter-based prognostics algorithm using matlab. An Dawn, Choi Joo-Ho, Kim Nam Ho. Reliability Engineering & System Safety. Elsevier. 2013;115;161-169.
- [42] Zio Enrico, Piloni Giovanni. Particle filtering prognostic estimation of the remaining useful life of nonlinear components. Reliability Engineering & System Safety. Elsevier. 2011;96(3):403-409.

- [43] Doucet Arnaud, Johansen Adam M. A tutorial on particle filtering and smoothing: Fifteen years later. *Hand of nonlinear filtering*. 2009;12(3):656-704.
- [44] Anishchenko Vadim S, Astakhov Vladimir, Neiman Alexander, Vadivasova Tatjana, Schimansky-Geier Lutz. *Nonlinear dynamics of chaotic and stochastic systems: tutorial and modern developments*. Springer Science & Business Media; 2007.
- [45] Casdagli Martin. A dynamical systems approach to modeling input-output systems. *Santa Fe Institute Studies in the Sciences of Complexity-Proceedings*, Addison-Wesley Publishing Co. 1992;12:265-265.
- [46] Gasperin M, Juricic D, Boskoski P, Vizintin J. Model-based prognostics of gear health using stochastic dynamical models. *Mechanical Systems and Signal Processing*. Elsevier. 2011;25(2):537-548.
- [47] Stochastic modeling of fatigue crack dynamics for on-line failure prognostics. Ray Asok, Tangirala Sekhar. *IEEE Transactions on Control Systems Technology*, IEEE. 1996;4(4):443-451.
- [48] Begg Colin D, Merdes Terri, Byington Carl, Maynard, Ken. *Dynamics modeling for mechanical fault diagnostics and prognostics*. Maintenance and Reliability Conf; 1999.
- [49] Okoh C, Roy R, Mehnen J, Redding L. Overview of remaining useful life prediction techniques in through-life engineering services. *Procedia CIRP*. Elsevier. 2014;16:158-163.
- [50] Biagetti Tatiana, Sciubba Enrico. *Automatic diagnostics and prognostics of energy conversion processes via knowledge-based systems*. Energy. Elsevier. 2004;29(12):2553-2572.
- [51] Schmoldt Daniel L, Rauscher H. Michael. *Building knowledge-based systems for natural resource management*. Chapman & Hall; 1996. .
- [52] Shrobe HE. *American Association for Artificial Intelligence. Exploring artificial intelligence: Survey talks from the national conferences on artificial intelligence*. Exploring artificial intelligence: survey talks from the National Conferences on Artificial Intelligence, Morgan Kaufmann Publishers; 1988.
- [53] Leondes Cornelius T. *Knowledge-based systems, four-volume set: techniques and applications*. Academic Press; 2000.
- [54] Leondes CT. *Intelligent Knowledge-based Systems: Knowledge-based systems. Intelligent Knowledge-based Systems: Business and Technology in the New Millennium*, Kluwer Academic; 2005
- [55] Leondes CT. *Expert systems: the technology of knowledge management and decision making for the 21st century. Expert Systems: The Technology of Knowledge Management and Decision Making for the 21st Century*. Academic Press; 2002.
- [56] Hunt VD. *Artificial intelligence & expert systems source*. Chapman and Hall Advanced Industrial Technology Series, Chapman & Hall; 1986.
- [57] Guida G, Tasso C. *Topics in expert system design: methodologies and tools. Studies in Computer Science and Artificial Intelligence*; 1989 North-Holland
- [58] Zadeh Lotfi A. *Fuzzy sets*. Information and Control. Elsevier. 1965;8(3):338-353.
- [59] *Fuzzy expert systems*. Kandel Abraham. CRC Press; 1991.
- [60] Zein-Sabatto Saleh, Bodruzzaman Jabir, Mikhail Mervat. *Statistical approach to online prognostics of turbine engine components*. Southeastcon, 2013 Proceedings of IEEE. 2013;1-6.
- [61] Swanson David C. *A general prognostic tracking algorithm for predictive maintenance*. Aerospace Conference, 2001, IEEE Proceedings. IEEE. 2001;6:2971-2977.
- [62] Line J. Kevin, Clements N. Scott. *A systematic approach for developing prognostic algorithms on large complex systems*. 2005 IEEE Aerospace Conference, IEEE. 2005;1-7.

- [63] Filev Dimitar P, Tseng Finn. Real time novelty detection modeling for machine health prognostics. NAFIPS 2006-2006 Annual Meeting of the North American Fuzzy Information Processing Society; 2006.
- [64] Chinnam Ratna Babu, Baruah Pundarikaksha. A neuro-fuzzy approach for estimating mean residual life in condition-based maintenance systems. *International Journal of Materials and Product Technology*, Inderscience Publishers. 2004;20(1-3):166-179.
- [65] Majidian A, Saidi MH. Comparison of fuzzy logic and neural network in life prediction of boiler tubes. *International Journal of Fatigue*. Elsevier. 2007;29(3):489-498.
- [66] Fuzzy sets, decision making, and expert systems. Zimmermann HJ. *International Series in Management Science Operations Research*; 1987. Springer Netherlands
- [67] Leondes Cornelius T. Fuzzy logic and expert systems applications. Academic Press. 1998;6.
- [68] Fuzzy expert systems and fuzzy reasoning. Siler William, Buckley James J. John Wiley & Sons; 2005.
- [69] Knowledge-based neurocomputing: A fuzzy logic approach. Kolman E, Margaliot M. *Studies in Fuzziness and Soft Computing*. Springer; 2009.
- [70] Schay G. Introduction to probability with statistical applications. Birkhäuser Boston; 2007. (ISBN 9780817645915, LCCN 2007930422)
- [71] Jones MC, Noufaily Angela, others. Log-location-scale-log-concave distributions for survival and reliability analysis. *The Institute of Mathematical Statistics and the Bernoulli Society Electronic Journal of Statistics*. 2015;9(2):2732-2750.
- [72] Mukhopadhyay Chiranjit, Roy Soumya. Bayesian accelerated life testing under competing log-location-scale family of causes of failure. *Computational Statistics*. Springer. 2016;31(1):89-119.
- [73] Survival analysis approach to reliability, survivability and Prognostics and Health Management (PHM). Ma Zhanshan Sam, Krings Axel W. *Aerospace Conference*, 2008 IEEE. 2008;1-20.
- [74] Gebraeel Nagi Z, Lawley Mark A, Li Rong, Ryan Jennifer K. Residual-life distributions from component degradation signals: A Bayesian approach. *IIE Transactions*. Taylor & Francis. 2005;37(6):543-557.
- [75] Ryan TP. *Modern engineering statistics*. Wiley; 2007. (ISBN 9780470128435)
- [76] Avionics health management: searching for the prognostics grail. Kirkland Larry V, Pombo Tony, Nelson Kody, Berghout Floyd. *Aerospace Conference*, 2004. *Proceedings*. 2004 IEEE. 2004;5:3448-3454.
- [77] Inman Daniel J, Farrar Charles R, Junior Vicente Lopes, Junior Valder Steffen. *Damage prognosis: for aerospace, civil and mechanical systems*. John Wiley & Sons; 2005.
- [78] Lovric M. *International encyclopedia of statistical science*. *International Encyclopedia of Statistical Science*. Springer Berlin Heidelberg; 2010. (ISBN 9783642048975, LCCN 2010937438)
- [79] Montgomery DC, Runger GC. *Applied statistics and probability for engineers*. John Wiley & Sons Incorporated; 2005.
- [80] Manly BFJ. *Multivariate statistical methods: A primer*, Third Edition. CRC Press; 2004. (ISBN 9781482285987)
- [81] Banerjee P. Pat, Avila Ruben, He David, Wu Shenliang, Bechhoefer Eric. Discriminant analysis based prognostics of avionic systems. *Systems, Man, and Cybernetics, Part C: Applications and Reviews*, IEEE Transactions on, IEEE. 2007;37(6):1318-1326.
- [82] Narsky Ilya, Porter Frank C. *Statistical analysis techniques in particle physics: Fits, density estimation and supervised learning*, John Wiley & Sons; 2013.
- [83] Chun Hyonho, Keles Sunduz. Sparse partial least squares regression for simultaneous dimension reduction and variable selection. *Journal of the Royal Statistical Society*:

- Series B (Statistical Methodology), Wiley Online Library. 2010;72(1):3-25.
- [84] Maitra Saikat, Yan Jun. Principle component analysis and partial least squares: Two dimension reduction techniques for regression. *Applying Multivariate Statistical Models*. 2008;79.
- [85] Meng Huixing, Li Yan-Fu. A review on prognostics and health management (PHM) methods of lithium-ion batteries. *Renewable and Sustainable Energy Reviews*. Elsevier. 2019;116;1-37.
- [86] Hamadache Moussa, Jung Joon Ha, Park Jungho, Youn Byeng D. A comprehensive review of artificial intelligence-based approaches for rolling element bearing PHM: shallow and deep learning. *JMST Advances*. Springer. 2019;1(1):125-151.
- [87] Ellefsen André Listou, Esøy Vilmar, Ushakov Sergey, Zhang Houxiang. A comprehensive survey of prognostics and health management based on deep learning for autonomous ships. *IEEE Transactions on Reliability*. IEEE. 2019;68(2):720-740.
- [88] Carvalho Thyago P, Soares Fabr´AAMN, Vita Roberto, Francisco Roberto da P, Basto João P, Alcalá Symone GS. A systematic literature review of machine learning methods applied to predictive maintenance. *Computers & Industrial Engineering*. Elsevier. 2019;137:106024.
- [89] Al-Mohamad Ahmad, Hoblos Ghaleb, Puig Vicenç. A hybrid system-level prognostics approach with online RUL forecasting for electronics-rich systems with unknown degradation behaviors. *Microelectronics Reliability*. Elsevier. 2020;111:113676.
- [90] Baur Marco, Albertelli Paolo, Monno Michele. A review of prognostics and health management of machine tools. *The International Journal of Advanced Manufacturing Technology*. Springer. 2020;107:2843-2863.
- [91] Liu Zepeng, Zhang Long. A review of failure modes, condition monitoring and fault diagnosis methods for large-scale wind turbine bearings. *Measurement*. Elsevier. 2020;149:107002.
- [92] Maldonado-Correa Jorge, Martín-Martínez Sergio, Artigao Estefanía, Gómez-Lázaro Emilio. Using SCADA data for wind turbine condition monitoring: a systematic literature review. *Energies. Multidisciplinary Digital Publishing Institute*. 2020;13(12):3132.
- [93] Rushdi Ali M, Bjaili Hasan A. Characterization of time to failure in prognostics: brief tutorial guide to prognostics professionals. *Journal of Advances in Mathematics and Computer Science*. 2017;1-15.
- [94] Bjaili Hasan A, Rushdi Ali M, Moinuddin Muhammad. Comparison of state-estimation algorithms for a noise-injected lithium-ion battery system. *British Journal of Mathematics & Computer Science*. 2017;22(6);1-13.

© 2020 Bjaili and Rushdi; This is an Open Access article distributed under the terms of the Creative Commons Attribution License (<http://creativecommons.org/licenses/by/4.0>), which permits unrestricted use, distribution and reproduction in any medium, provided the original work is properly cited.

Peer-review history:
The peer review history for this paper can be accessed here:
<http://www.sdiarticle4.com/review-history/62205>