



# **Application of the Two-parameter Weibull Distribution Method to Assess the Reliability of Gas Turbine Compressors**

**Sunday N. Ozonuwe<sup>1\*</sup>, Dennar Onyekachi<sup>1</sup> and Collins U. Oside<sup>2</sup>**

<sup>1</sup>*Department of Mechanical Engineering, Petroleum Training Institute, Warri, Nigeria.*

<sup>2</sup>*Department of Mechanical Engineering Technology, National Institute of Construction Technology and Management, Uromi, Nigeria.*

## **Authors' contributions**

*This work was carried out in collaboration among all authors. Author SNO designed the study, performed the statistical analysis, wrote the protocol and wrote the first draft of the manuscript. Authors DO and CUO managed the literature searches and analyses of the study. All authors read and approved the final manuscript.*

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## **ABSTRACT**

In this paper, the reliability of axial compressors of gas turbines used in oil and gas installations was analyzed by the application of the two-parameter Weibull distribution method. Reliability analysis is an important tool to minimize the failures of equipment, shorten repair time and enhance maintenance effectiveness because when a system is well-maintained system, it minimizes operating costs and optimizes throughput. The study involves the analysis of the failure times of three identical gas turbine compressors. The two-parameter Weibull distribution method was found to best fit the failure data with a correlation coefficient of 0.9886 using the rank regression method. Based on the result obtained for the two-parameter Weibull estimates, it is deduced that the compressors under consideration are at the end of their operational life (wear-out period). From the Weibull estimates obtained, the reliability and mean time to failure of the compressors were determined. This result can be used to carry out timely and effective maintenance of the compressors so as to improve their operation and availability.

\*Corresponding author: Email: [idiaphoken@gmail.com](mailto:idiaphoken@gmail.com);

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## 1. INTRODUCTION

Nowadays, gas turbines are one of the most widely used power generating technologies. They find applications in power generation, as mechanical drives in industries and majorly in aviation as propulsion systems on air, land and sea [1-3]. The compressor unit is one of the three critical components of the gas turbine along with the combustion chamber and turbine. The failure or malfunctioning of the compressor will lead to total failure of the gas turbine, hence, the need to ensure its safe working conditions [4,5]. The focus of this study is to investigate the failure of three identical 32 MW gas turbine axial compressor used in oil and gas installations for power generation by the application of the two-parameter Weibull distribution with a view to minimizing these failure risks through activities such as reliability evaluation, regular maintenance actions and performance monitoring of the equipment in order to maintain the reliability and availability of the compressors. Reliability analysis methods have been accepted as standard tools for the planning, design, operation, and maintenance of gas turbine plants. As a result, the distribution parameters can be used to estimate important life characteristics such as mean time to failure, hazard rate, reliability and probability of failure at a given time [6,7]. Researchers such as Agnihotri et al. [8] have applied reliability analysis to assess the reliability of a system of boilers used in readymade garments industry. Gupta et al. [9] have also studied the reliability and availability analysis in a cement manufacturing industry and a butter oil manufacturing industry. Lai et al. [10] have performed reliability and availability analysis for some complex systems in a software/hardware industry. Gupta et al. [11] have discussed the reliability analysis of a butter oil manufacturing process in a dairy milk plant by considering the exponential distribution of failure rates for various components. Kaur [12] has analyzed the reliability, availability and maintainability (RAM) analysis of an industrial process. Sachdeva et al. [13] have proposed a reliability analysis for a pulping system in a paper industry. [14-16] have discussed reliability and availability analysis of complex engineering systems. The Weibull distribution is used to model extreme value data. One reason it is widely used in reliability and life data analysis is due to its flexibility. It can mimic various

distributions like the normal. One of such applications is the frequent use of the Weibull distribution to model failure time data Murthy et al. [17]. Another discipline where the Weibull is applicable is in the analysis of wind speeds. In the past, the two-parameter Weibull distribution has been shown to effectively describe the variation of wind speed and is commonly used in modeling such data [18,19]. Other researchers such as Krantz [20] successfully modeled gear fatigue test data by applying the two-parameter Weibull distribution. Malgorzata and Bogna [21] discussed the evaluation bond strength of resin cements using the two-parameter Weibull distribution. The study evaluated and compared Weibull parameters of resin bond strength values by using six different general-purpose statistical software packages. Shazaib et al. [22] carried out a study to the analysis the reliability of gas turbine engine by means of bathtub-shaped failure rate distribution. The results obtained from the study showed the effectiveness of the method for assessing operational reliability. Djeddi et al. [23] discussed operational reliability analysis applied to a gas turbine based on three-parameter Weibull distribution. The study focused on the problems of reliability analysis in industrial equipment. Ronald H. Salzman [24] applied Weibull distribution methods to study failure in gas turbine component. In this paper, the two-parameter Weibull distribution is used to model the failure time of three identical gas turbine compressors by using the rank regression estimation method. The estimation of the Weibull parameters was done with statistical software Windchill quality solutions 11 and the mean time to failure (MTTF), failure rate and the reliability of the compressors were successfully determined.

## 2. METHODOLOGY

Three 32 MW gas turbines used in an oil and gas company located in Forcados were used for this study. Data containing failure times of three compressors during the operating period of 2005 to 2015 was obtained from maintenance records of the company. This data was analyzed and modeled using two-parameter Weibull distribution through the application of a special reliability software package. The rank regression estimation method is employed to obtain estimates of the Weibull distribution. In order to apply the rank regression estimation method, the

failure times were ranked in order of increasing magnitude; from smallest to highest. There are many distributions that can be used to model failure data, such as the normal distribution, exponential distribution, Rayleigh distribution, Weibull distribution, Gamma distribution and others. The main challenge of fitting distributions to reliability data is finding the type of distribution and the values of the parameters that give the highest probability of producing the observed data. One of the most common probability density functions used in industry is the Weibull distribution. Weibull distribution can be applied to a large number of situations. It is also used to model both increasing and decreasing failure rates. Weibull analysis involves using the Weibull distribution to study the analysis of time to failure. The main advantage of using this distribution is its ability to handle small samples of failure data and its flexibility in fitting different failure modes. It also produces an easy-to-understand plot. Small samples are very common in reliability testing because tests are often destructive in nature and also very costly. Weibull can be fitted in two ways: (i) two-parameter or (ii) three-parameter distributions. In this paper, the two-parameter distribution is chosen over the three-parameter because its application is more common and very efficient when dealing the small sample size.

The two-parameter Weibull distribution has the scale parameter ( $\eta$ ) and shape parameter ( $\beta$ ), while the location parameter is assumed to be zero. The values of the shape parameter ( $\beta$ ) determine the shape of the distribution. If  $\beta < 1$ , it means the failure rate is decreasing, meaning there is high probability of failing at early stages (infant mortality). If  $\beta = 1$ , it means the failures are independent of time (constant failure rate). If  $1 < \beta < 4$ , it means the failure rate is increasing. This is also known as the early wear out period which can be due to generic failure modes, such as corrosion, aging or friction. When  $\beta > 4$ , it means rapid wear out, meaning steep curve with fast wear out at some point. In the past, techniques to perform Weibull analysis by hand were tedious and less accurate. These days, the process has been replaced by specialized Weibull analysis software programs that can accurately estimate the Weibull parameters and also automatically produce Weibull plots from a given data sets. In this study, TPC Windchill Quality Solutions 11.0 software package is used to determine the estimates of two-parameter Weibull distribution ( $\beta$  and  $\eta$ ). When the estimates of the two-parameter Weibull

distribution are accurately obtained, they can be used to assess the mean time to failure and reliability of the gas turbine compressors.

The following equations are used to determine the mean time to failure (MTTF) and reliability of two-parameter Weibull distribution.

The mean time to failure (MTTF) is obtained using:

$$MTTF = \eta \Gamma(1 + \frac{1}{\beta}) \tag{1}$$

The failure rate function (also known as the hazard rate) is:

$$\lambda(t) = \frac{\beta}{\eta} (\frac{t}{\eta})^{\beta-1} \tag{2}$$

The reliability function of the Weibull distribution is given by:

$$R(t) = e^{- (\frac{t}{\eta})^{\beta}} \tag{3}$$

Where  $t, \beta, \eta > 0$

$$\Gamma(x) = \int_0^{\infty} e^{-x} x^{n-1} \tag{4}$$

Where  $\Gamma(x)$  = Gamma Function  
 $\eta$  = Scale parameter  
 $\beta$  = Shape parameter  
 $t$  = Time to failure

## 2.1 Weibull Parameters Estimation

There are several methods of estimating Weibull parameters, such as the maximum likelihood estimation, method of moment and median rank regression. Olteanu and Freeman [25] studied the performance of maximum likelihood estimation (MLE) and median rank regression (MRR) methods and they concluded that the median rank regression method offered the best combination of accuracy and ease of interpretation when the sample size and number of suspensions are small. This method is very popular in industry because fitting can be easily visualized. The median rank regression method is therefore used in this study because it will offer the best fit for our small sample size. Furthermore, this technique provides a good measure of the goodness-of-fit of the chosen distribution in the correlation coefficient. The rank regression method determines the way estimated unreliabilities are associated with failure times under consideration. The median rank method

assigns unreliability estimates based on the failure order number and the cumulative binomial distribution.

In the rank regression method, the Weibull shape and scale parameters ( $\beta$  and  $\eta$ ) are determined using the following equations:

$$\beta = \left[ \frac{n(\sum xy) - (\sum x)(\sum y)}{n(\sum x^2) - (\sum x)^2} \right] \tag{5}$$

$$\ln \eta = \left[ \frac{(\sum y) - \beta(\sum x)}{n} \right] \tag{6}$$

One of the advantages of the rank regression method is that it can provide a good measure for the fit of the line to the data points. This measure is known as the correlation coefficient,  $\rho$ . In the case of life data analysis, it is a measure for the strength of the linear relation between the median ranks (y-axis values) and the failure time data (x-axis values). The correlation coefficient has the following form:

$$\rho = \frac{\sigma_{xy}}{\sigma_x \sigma_y} \tag{7}$$

Where  $\sigma_{xy}$  is the covariance of  $x$  and  $y$ ,  $\sigma_x$  is the standard deviation of  $x$ , and  $\sigma_y$  is the standard deviation of  $y$ . The closer the value of the correlation coefficient,  $\rho$ , is to the absolute value of 1, the better the linear fit. Note that when  $\rho = +1$  indicates a perfect fit with a positive slope, while  $-1$  indicates a perfect fit with a negative slope. A perfect fit means that all of the points fall exactly on a straight line. A correlation coefficient value of zero would indicate that the data points are randomly scattered and have no pattern or correlation in relation to the regression line model.

### 3. RESULTS AND DISCUSSION

#### 3.1 Results

The failure times of three gas turbine axial compressors used in oil and gas installations is shown in Table 1. In order to apply the rank regression method, the data in Table 1 were arranged in increasing order of magnitude (from smallest to largest). The smallest value (815) is assigned the rank of 1, followed by the second smallest value (817) and so forth to the highest value (3296) ranked 20. These ranked data is presented in Table 2.

#### 3.2 Analysis of Gas Turbine Axial Compressor

The data containing the failure times of three gas turbine axial compressors in Table 2 is analyzed

using two-parameter Weibull distribution through the application of reliability analysis software Windchill quality solutions 11 to obtain the estimates of the Weibull distribution, namely, scale parameter and shape parameter and the coefficient of correlation,  $r$ . The rank regression estimation method was employed in the analysis and the result presented in Table 3. Figs. 1, 2, 3 and 4 show the probability plot; the probability density function (PDF) plot, plot of failure rate against time and the reliability plot of the Weibull distribution respectively. All the figures were obtained automatically using Windchill quality solutions software.

**Table 1. Raw data of failure time (hours) of gas turbine compressor**

1852	1050
2051	815
1453	2252
2170	3296
1593	1305
2536	817
1218	1414
1654	2365
3058	2601
2578	1811

**Table 2. Ranked Failure time of gas turbine compressor**

Rank	Failure time (hour)
1	815
2	817
3	1050
4	1218
5	1305
6	1414
7	1453
8	1593
9	1654
10	1811
11	1852
12	2051
13	2170
14	2252
15	2365
16	2536
17	2578
18	2601
19	3058
20	3296

The MTTF, failure rate  $\lambda(t)$  and reliability of the gas turbine compressors can be obtained by applying the results obtained in our analysis,

where  
 $\beta = 2.9349$ ,  
 $\eta = 2120.5869$  and time to failure,  $t = 1142\text{hr}$

From equation (1), the mean time to failure of the two-parameter Weibull distribution is calculated as follows:

$$\begin{aligned}
 MTTF &= \eta \Gamma \left( 1 + \frac{1}{\beta} \right) \\
 MTTF &= 2120.5869 \times \Gamma \left( 1 + \frac{1}{2.9349} \right) \\
 MTTF &= 2120.5869 \times \Gamma (1.34) \\
 \text{Using the gamma functions table, } \Gamma(1.34) &= 0.8922 \\
 MTTF &= 2120.5869 \times 0.8922 \\
 MTTF &= 1891.9876\text{hr}
 \end{aligned}
 \tag{8}$$

The failure rate of the two-parameter Weibull distribution is obtained using equation (2)

$$\begin{aligned}
 \lambda(t) &= \frac{2.9349}{2120.5869} \left( \frac{1142}{2120.5869} \right)^{2.9349-1} \\
 \lambda(t) &= \frac{2.9349}{2120.5869} \left( \frac{1142}{2120.5869} \right)^{1.9349} \\
 \lambda(t) &= 0.001384(0.5385)^{1.9349} \\
 \lambda(t) &= 0.001384 \times 0.3019 \\
 \lambda(t) &= 0.0004178/\text{hr}
 \end{aligned}
 \tag{9}$$

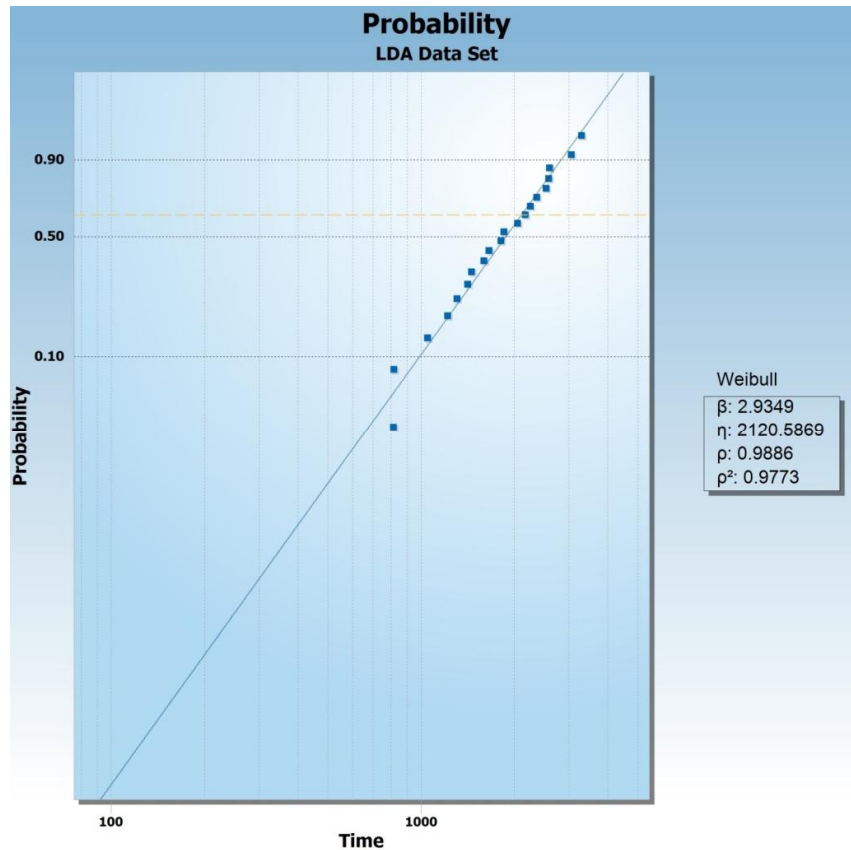
The failure rate of the three identical gas turbine compressors is estimated to be 0.0004178/hr.

Applying equation (3), the reliability is calculated as follows:

$$\begin{aligned}
 R(t) &= e^{-\left( \frac{1142}{2120.5869} \right)^{2.9349}} \\
 R(t) &= e^{-(0.5385)^{2.9349}} \\
 R(t) &= e^{-0.1626} \\
 R(t) &= 0.8499
 \end{aligned}
 \tag{10}$$

**Table 3. Two-parameter Weibull estimates**

Shape parameter, $\beta$	Scale parameter, $\eta$	Coefficient of correlation, $\rho$
2.9349	2120.5869	0.9886



**Fig. 1. Probability plot**

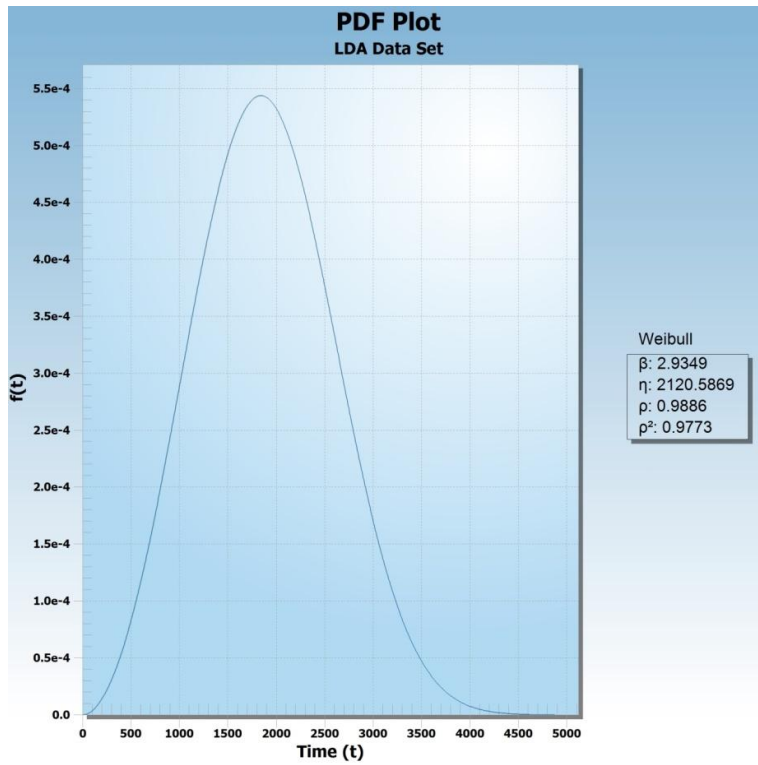


Fig. 2. Plot of PDF against time

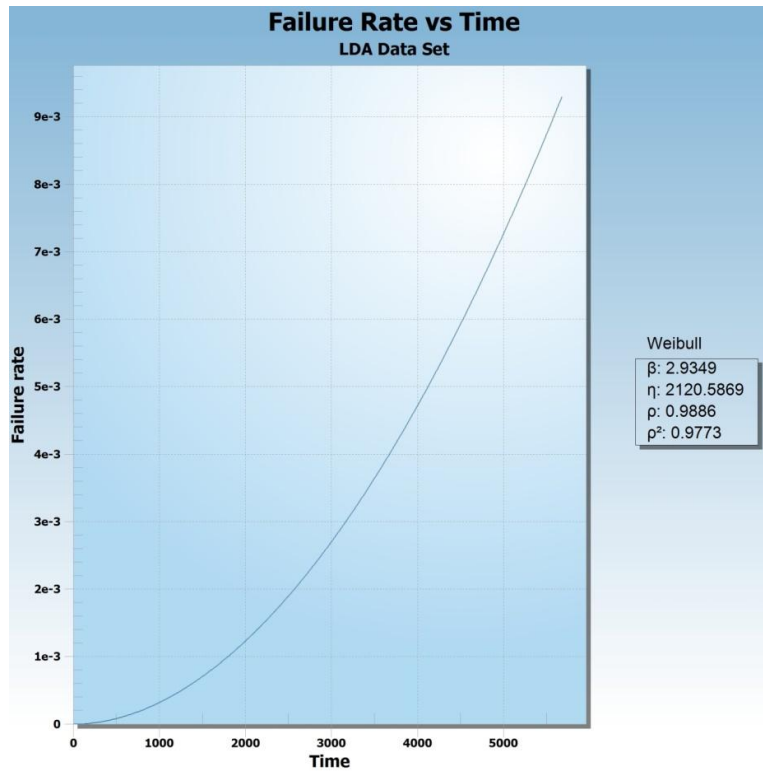


Fig. 3. Failure rate against time

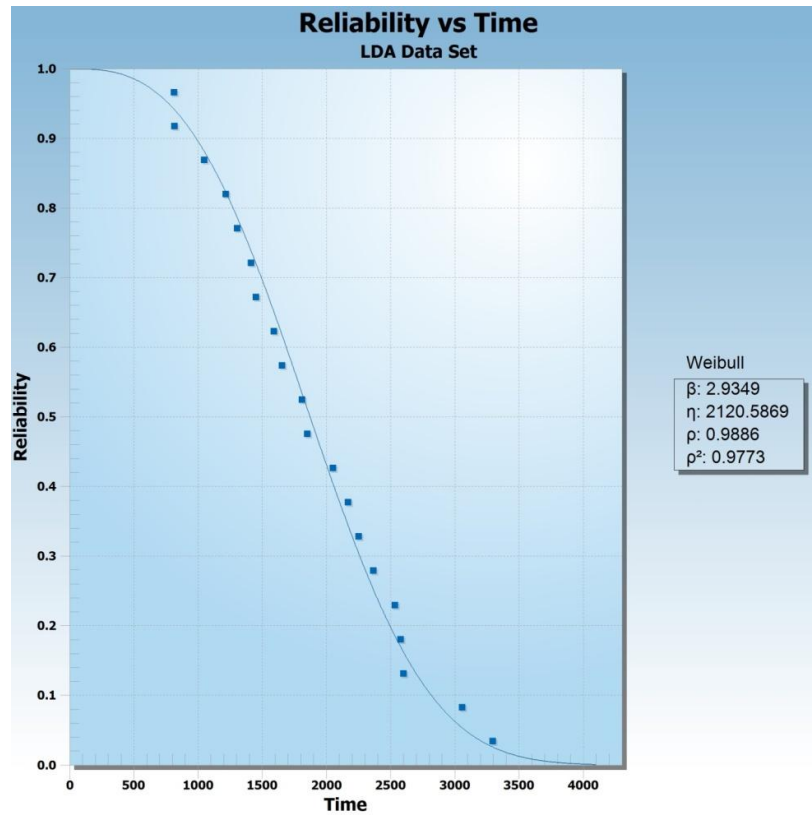


Fig. 4. Plot of reliability against time

### 3.3 Discussion

In this study, the two-parameter Weibull distribution using the median rank regression estimation method is used to analyze the failure times of three identical 32MW gas turbine axial compressors. Fig. 1 shows the probability of failure over time. From the plot, it is obvious that the Weibull distribution function  $F(t)$  is a linear function of  $t$ . This means that our data was adequately described by the two-parameter Weibull distribution. Fig. 2 shows the plot of the two-parameter probability density function (PDF) over time,  $t$ . From the plot, the shape of the probability density function of the Weibull distribution is observed to be symmetrical and bell-shaped, just like the normal distribution. The probability density function steadily increases until it gets to its mode at time,  $t=1,750$  hr, after that point, it can be seen to be decreasing gradually. The plot of the failure rate against time is shown in Fig. 3. In Fig. 3, the failure rate of the Weibull distribution can be seen increasing over time,  $t$ , which is indicative that the compressors are in their early wear-out periods. At this point, the compressors are beginning to wear and tear

as a result of ageing. In Fig. 4, the reliability plot of the two-parameter Weibull distribution is shown. From the plot, a gradual drop in reliability is observed as the time to failure increases. It is a well known fact that no matter how well a component or system is designed or manufactured; it is bound to fail at some point during its operating life. The drop in the reliability of the gas turbine compressors is therefore expected as they have been in operation for a long time and ageing is beginning to occur.

The estimates of the two-parameter Weibull distribution, namely the shape and scale parameter ( $\beta$  and  $\eta$ ) and the correlation coefficient,  $\rho$ , were determined using reliability software, TPC Windchill quality solutions 11. The following results were obtained:  $\beta = 2.9349, \eta = 2120.5869$  and  $\rho = 0.9886$  as shown in Table 3. The value of correlation coefficient  $\rho = 0.9886$ , shows that there is a good linear fit for the Weibull distribution. This means the rank regression method provided a good measure of fit as a strong correlation is observed between the data points. From the result obtained for the shape parameter, it is obvious that the three gas

turbine compressors are in their early wear-out period as  $\beta > 1$ . This also means that the failure rate of the compressors is gradually increasing as the disks are continuing to wear due to ageing, friction and corrosion. The values of the shape and scale parameters obtained in our analysis were used to compute the mean time to failure (MTTF), failure rate and reliability of the gas turbine compressors using equations (1-3) and the following results were obtained:  $MTTF = 1892 \text{ hr}$ ,  $\lambda(t) = 0.0004198$  and  $R(t) = 0.8499$ .

#### 4. CONCLUSION

The application of the two-parameter Weibull distribution method to assess the reliability of three identical 32 MW gas turbine compressors has been investigated. The study involves the analysis of the failure times of the compressors using the median rank regression estimation method. Their failure times were rank-ordered and estimates of the two-parameter Weibull distribution; shape and scale parameters were determined using TPC Windchill quality solutions 11. This method of estimation was found to be very fast and more accurate than the manual approach. The value of the correlation coefficient obtained in the study showed that the two-parameter Weibull distribution was suitable for fitting the data points. The results of our analysis showed that the gas turbine compressors are in their early wear-out period as their failure rate was observed to be increasing steadily over time and the value of the shape parameter,  $\beta = 2.9349$  obtained is greater than 1. The MTTF, failure rate and reliability were successfully computed using the values of the shape and scale parameters obtained. The reliability of the gas turbine axial compressors was found to be about 85%.

#### COMPETING INTERESTS

Authors have declared that no competing interests exist.

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