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Modeling the Failure Rate of a Standby Multi-Component System and Improving Reliability

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Abstract

Evaluating the reliability of a system requires knowledge of the failure modes to which it is subjected. Complex topology systems generally require a high level of availability, which is a function of the arrangement of elements (components) in the system. To avoid serious failures for such complex systems, recourse can be had to the redundancy techniques available in the literature. These techniques help to improve system reliability, without affecting the reliability of system components. This paper is interested in the proposal of a model for evaluating the failure rate of a standby multi-components system and in improving the reliability of mechanical systems, arranged in a topology (series, parallel, or mixed).

Keywords

Reliability, Modeling, Topology, Redundancy, Availability, Failure Rate

1. Introduction

The shutdown of a production equipment in operation, following a failure, causes great tensions in the production actor. Indeed, there is a loss of time, a loss in production, therefore of money, then a loss of credibility with the customers of the company, therefore loss of money again etc. It is an infernal circle in which, as we can easily understand, no industrialist wishes to enter, hence the importance that must be given to the very high demand for the evaluation of reliability in accordance with the safety standard of operation. The objective of reliability methods is to assess the probability of failure of a physical system in relation to a given failure scenario, taking into account the uncertainties in the description of the model. Several authors have defined reliability in the literature. Reliability is the ability of an entity to perform a required function or meet user needs, under given conditions, for a given period of time [1] [2] [3].

As the complexity of a mechanical system increases, its reliability decreases if compensatory measures are not taken [4] [5]. For a system made up of independent components, that is to say in series, if only one of the components has failed, the system no longer works. It is said that the system is without redundancy. The reliability of this system deteriorates dramatically with the increase in the number of components.

Figure 1 illustrates the curves showing the reliability of the system as a function of the number of components (each component *i* having a reliability R_i).

As a result, the reliability of the system (R_{ss}) decreases when the number *k* of elements increases, particularly for systems in which the components do not have very high individual reliability.

Several works in the literature have focused on the modeling of reliability, like those of [6] [7] [8]. It happens that in his work, we do not develop models allowing to evaluate the failure rate of *n*-component systems in standby. In the following sections, we will present the different models of reliability improvement by redundancies and to conjecture by an application a model of evaluation of the rate of failures of the systems with *n* components in standby and to make a comparative study of the improvement of the reliability by active and passive redundancies.

2. Improved Reliability through Redundancies

Redundancy is very widespread in areas where dependability is crucial for the safety of people and the environment, such as aeronautics or nuclear. Generally speaking, real systems are made up of several components and have several failure modes, such systems are said to be complex and their analysis becomes increasingly difficult. Integrating redundancy into systems is particularly effective

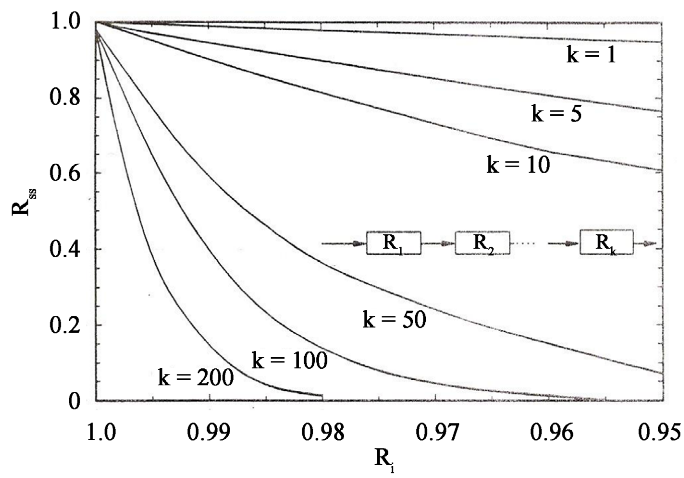

Figure 1. Reliability of a series system according to the number of components.

when random failures predominate or in critical systems. This suggests that such a technique contributes to increased reliability [9] [10] [11] [12] [13].

A redundant system contains one or more components or subsystems of standby in the system configuration. These standby units will allow the system to continue operating when the main unit fails. System failure only occurs when all or some of the standby units fail. Therefore, redundancy is a system design technique that can increase the reliability of the system. This application aims to increase the total reliability of the system by a parallel arrangement of components of different reliability. Figure 2 shows the improvement in system reliability based on the number of components and their reliability. However, this approach remains expensive for low complexity systems.

Redundancy therefore consists in having several copies of the same equipment or the same process or any other element participating in a mechanical, electronic or industrial solution [15].

Depending on the circumstances, it is useful:

- to increase the total capacity or performance of a system,
- to reduce the risk of breakdown,
- to combine these two effects.

The problem with reliability maintenance is its constant improvement. It can therefore intervene on the component technology, arrange the components or subsystems in such a way as to make them more reliable by the use of redundancies, of which there are 3 main categories:

- Active redundancies,
- Passive redundancy or "stand-by",
- Majority redundancies.

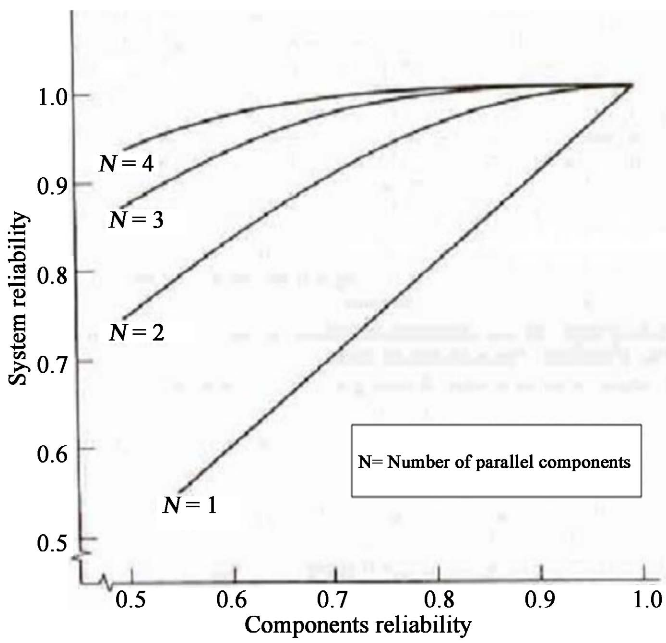

Figure 2. The reliability of the system as a function of the number of components and their reliability [14].

2.1. Active Redundancy

Active redundancy is achieved by paralleling elements providing the same functions and working at the same time. We are therefore dealing with a system called by the Reliability experts "parallel topology system".

A distinction is made between total and partial active redundancy. **Figure 3** diagrams a system with total active redundancy which only becomes faulty with the failure of the last surviving element. By definition, it is a system in which the elements are associated in parallel.

For n independent components in active redundancy, the system reliability law is determined by Equation (1).

$$R_{sys} = P\left(E_1 \cup E_2 \cup \cdots \cup E_n\right) \quad (1)$$

By applying Henri Poincaré's formula, in the case of three independent components with active redundancy, the law of reliability according to an exponential law is given by the relation (2).

$$R\left(t\right) = \mathrm{e}^{-\lambda_1 t} + \mathrm{e}^{-\lambda_2 t} + \mathrm{e}^{-\lambda_3 t} - \mathrm{e}^{-(\lambda_1+\lambda_2)t} - \mathrm{e}^{-(\lambda_1+\lambda_3)t} - \mathrm{e}^{-(\lambda_2+\lambda_3)t} + \mathrm{e}^{-(\lambda_1+\lambda_2+\lambda_3)t} \quad (2)$$

If the failure rates are equal, the system failure rate is given by the relation (3).

$$\lambda\left(t\right) = \lambda \frac{0.5 - \mathrm{e}^{-\lambda t} + 0.5\mathrm{e}^{-2\lambda t}}{0.5 - 0.5\mathrm{e}^{-\lambda t} + \frac{1}{6}\mathrm{e}^{-2\lambda t}} \quad (3)$$

We speak of partial active redundancy when a system has n elements, of which k ($k < n$) is strictly necessary for it to function. The system can therefore accept ($n - k$) failures.

2.2. Passive Redundancy

Redundancy is said to be passive or (standby) when the superabundant elements are not put into service until the time of need. In this case, only one element works, the others are pending. This has the advantage of reducing or eliminating the aging of non-working elements. On the other hand, there is the disadvantage of being obliged to have a fault detection and switching device for a system. The diagram for passive redundancy is given in **Figure 4**.

Schematically, it involves a switching detection (DC) detecting the commissioning of the standby element when the main component fails. The expressions for calculating reliability for such systems have been established [16] [17].

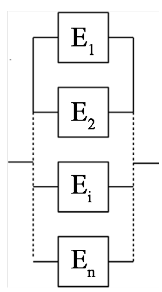

Figure 3. Total active redundancy.

In this case, there are two possibilities:

1) The main component does not fail, $t_1 \succ t$.

2) The main component fails but the standby component does not fail $t_1 \prec t$ and $t_2 \succ t$.

Since these two possibilities are mutually exclusive, the probabilities can be added together. The reliability of the passive redundancy system is therefore calculated by Equation (4).

$$R_{sys} = R(t_1) - \int_0^t R_2(t - t_2) \cdot \frac{\mathrm{d}}{\mathrm{d}t} R_1(t) \cdot \mathrm{d}t \tag{4}$$

In the case where we have two components with λ identical and constant, the reliability is given by the relation (5).

$$R_{sys}(t) = \mathrm{e}^{-\lambda t} \cdot (1 + \lambda t) \tag{5}$$

The instantaneous failure rate, in passive redundancy, in the case of two components with identical and constant λ, is calculated by Equation (6).

$$\lambda(t) = \frac{\lambda^2 t}{1 + \lambda t} \tag{6}$$

In the case of several components waiting (standby) or queued, each subsystem is identical and subject to the exponential law as shown in **Figure 5**, the reliability of the system is calculated by the relation (7).

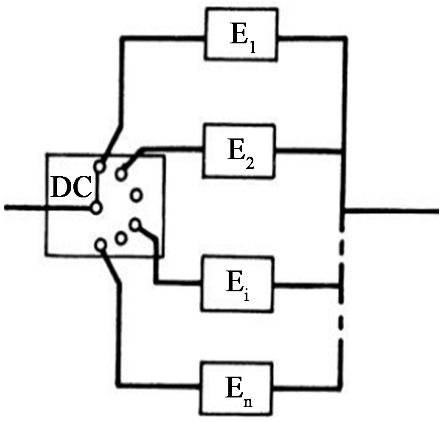

Figure 4. Passive redundancy.

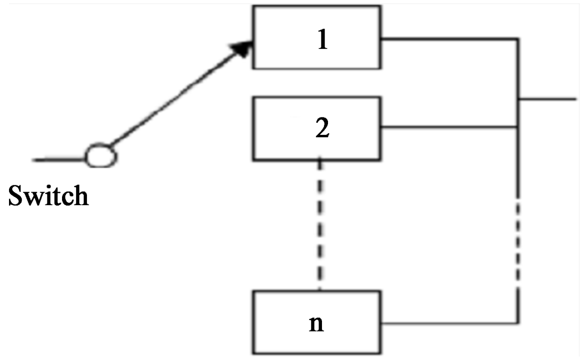

Figure 5. Several components waiting.

$$R_{sys}(t) = \mathrm{e}^{-\lambda t}\sum_{j=0}^{n-1}\frac{(\lambda t)^j}{j!} \tag{7}$$

The reliability is calculated according to a fish law of average λt. It is equal to the probability that all components except one.

The mean of the law of fish λt is equal to the average number of defaulters at time t.

Relation (7) will be considered to calculate the reliability of multi-component standby systems. This shows that the arrangement of the components (in active or passive redundancy) in a production chain directly influences the reliability and therefore the availability of the production tool. However, the work of [10] [11] presents a limit insofar as they do not propose a model for evaluating the failure rate of systems with several components in standby. In the application section, we will propose a model for assessing the instantaneous failure rate of a standby multi-component system.

2.3. Majority Redundancy

The majority redundancy is such that the function is ensured if at least the majority of the elements are in working order.

This redundancy mainly concerns high security signals, and in particular electronic equipment. The output signal is that of most components. The simplest case has 3 elements.

Figure 6 shows a situation of majority redundancy.

We consider that the decision-making organ D has a reliability equal to 1.

Let R_{sys} be the probability of having more than 2 elements in correct operation.

If $R_{E_1} = R_{E_2} = R_{E_3} = R$, we have:

$$R_{sys} = \sum_{k=2}^{3} C_3^k R.^k (1-R)^{3-k} = 3R^2 - 2R^3 \tag{8}$$

If we generalize to n (obligatorily odd to have a majority) elements, we obtain:

$$R_{sys} = \sum_{k=c}^{n} C_n^k R.^k (1-R)^{n-k} \quad \text{with} \quad c = \frac{n+1}{2} \tag{9}$$

The formula for calculating "c" provides the majority of the elements.

Taking into account the reliability of the decision component:

$$R_{sys} = R_D \sum_{k=c}^{n} C_n^k R.^k (1-R)^{n-k} \quad \text{with} \quad c = \frac{n+1}{2} \tag{10}$$

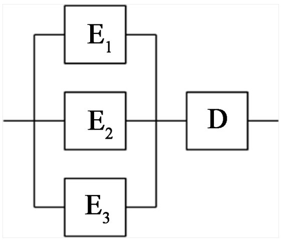

Figure 6. Majority redundancy.

3. Applications

3.1. Improving the Reliability of a Serial Topology System

Consider the process of a production chain, consisting of 8 units, shown in Figure 7.

The basic reliability of the units is as follows: $R_1 = 0.84$,
$R_2 = R_3 = R_4 = R_5 = R_7 = R_8 = 0.98$ and $R_6 = 0.7$.

The reliability of the system is given by the relation (11).

$$R_{sys} = \prod_{i=1}^{8} R_i \tag{11}$$

By applying this relation, we find: $R_{sys} = 0.53$.

To improve this reliability, redundancies can be applied to the least reliable units: E_1 and E_6 (Figure 8).

The equivalent reliability of the proposed new topology system is worth:

$$R'_{sys} = \left[1-(1-0.84)^2\right] \times 0.98^4 \times \left[1-(1-0.7)^3\right] \times 0.98^2 = 0.84$$

We note that $R'_{sys} > R_{sys}$, from where we have a satisfactory result, because the proposed topology improves reliability by 58.5%.

3.2. Improving the Reliability of a System by Active and Passive Redundancy

Considering a system with three components in redundancy, the failure rate $\lambda = 5.40 \times 10^{-5}$/hour, $t = 6$ years without maintenance, we determine the reliability of the system and its failure rate for different times of mission (1 to 6 years).

3.2.1. System Operation in Active Redundancy

When the system operates in active redundancy, the reliability and the failure rate are given by relations (2) and (3).

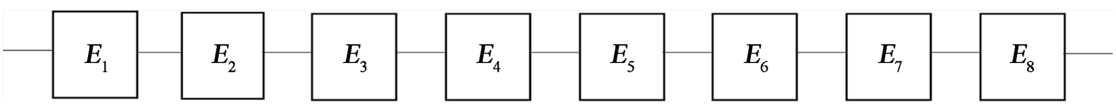

Figure 7. Process of a production chain.

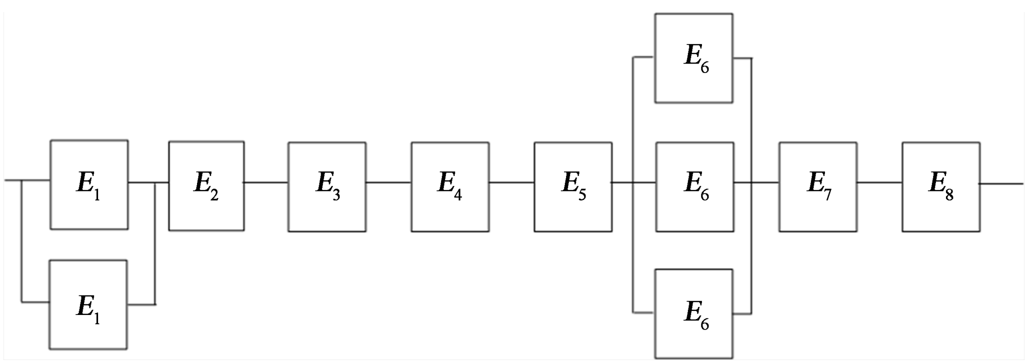

Figure 8. Proposed topology.

Figure 9 shows the evolution of the reliability of the system as a function of the service time.

It can be seen that the reliability of the system decreases drastically over time. The optimum reliability for this system is worth 0.9906.

Figure 10 shows the variation in the failure rate of the redundant system of the three components in parallel.

The failure rate is growing exponentially. The more time increases, the closer we get to the old age phase.

3.2.2. System Operation in Passive Redundancy

When the system operates in passive redundancy, reliability is given by equation (7). In our case with three components mounted in parallel, the relation (7) becomes the relation (12).

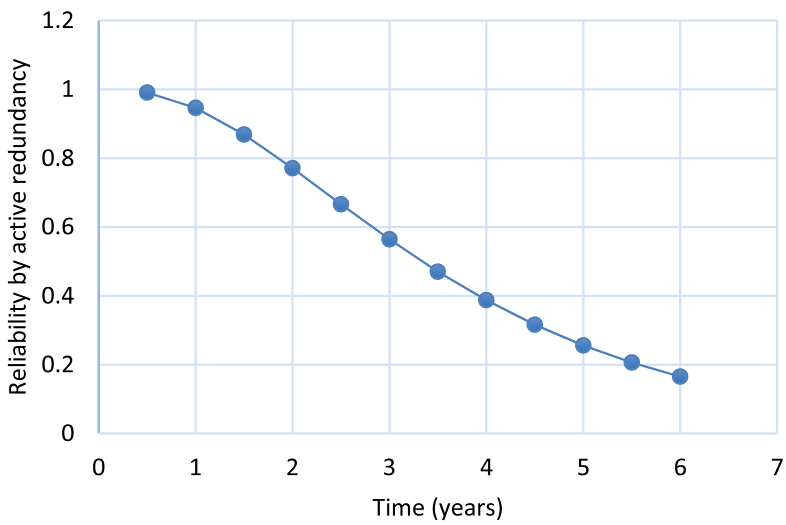

Figure 9. Variation in system reliability by active redundancy.

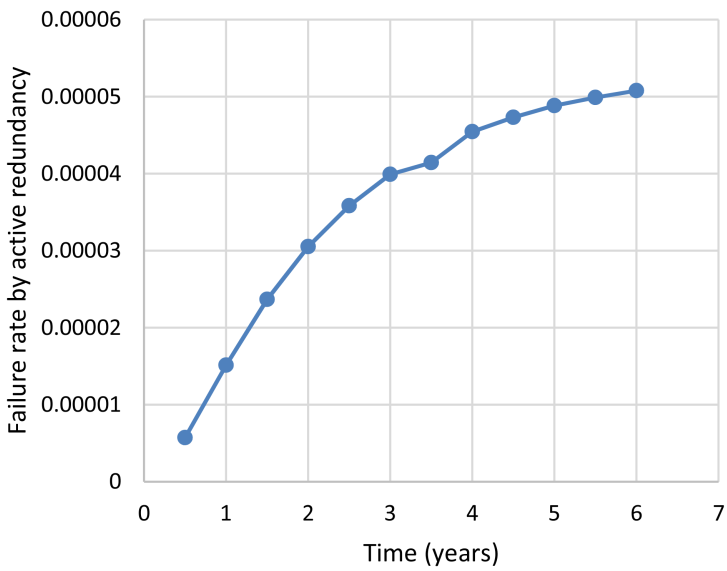

Figure 10. Variation in failure rate by active redundancy.

$$R_{sys} = e^{-\lambda t}\sum_{j=0}^{2}\frac{(\lambda t)^j}{j!} = e^{-\lambda t}\left(1+\lambda t+\frac{(\lambda t)^2}{2}\right) \quad (12)$$

However, what about the failure rate assessment relationship in this case?

We will therefore propose a model to assess the failure rate of a standby multi-component system.

By definition, we have: $\lambda(t) = -\frac{1}{R_{sys}(t)}\frac{dR_{sys}(t)}{dt}$

Now, $R_{sys}(t) = e^{-\lambda t}\sum_{j=0}^{n-1}\frac{(\lambda t)^j}{j!}$

So: $\frac{dR_{sys}(t)}{dt} = \frac{d}{dt}\left(e^{-\lambda t}\sum_{j=0}^{n-1}\frac{(\lambda t)^j}{j!}\right) = \frac{d}{dt}e^{-\lambda t}\cdot\sum_{j=0}^{n-1}\frac{(\lambda t)^j}{j!} + \frac{d}{dt}\sum_{j=0}^{n-1}\frac{(\lambda t)^j}{j!}\cdot e^{-\lambda t}$

Also, $\frac{d}{dt}\sum_{j=0}^{n-1}\frac{(\lambda t)^j}{j!} = \sum_{j=0}^{n-1}\frac{d}{dt}\frac{(\lambda t)^j}{j!} = \sum_{j=0}^{n-1}\frac{\lambda^j\cdot t^{j-1}}{(j-1)!}$

So: $\frac{dR_{sys}(t)}{dt} = e^{-\lambda t}\left(\sum_{j=0}^{n-1}\frac{\lambda^j\cdot t^{j-1}}{(j-1)!} - \lambda\sum_{j=0}^{n-1}\frac{(\lambda t)^j}{j!}\right) = e^{-\lambda t}\sum_{j=0}^{n-1}\left(\frac{\lambda^j\cdot t^{j-1}}{(j-1)!} - \lambda\frac{(\lambda t)^j}{j!}\right)$

$$\text{Hence: } \lambda(t) = \frac{\sum_{j=0}^{n-1}\frac{(\lambda t)^j}{j!}\left(\lambda-\frac{j}{t}\right)}{\sum_{j=0}^{n-1}\frac{(\lambda t)^j}{j!}} \quad (13)$$

The model for evaluating the failure rate of a standby multi-component system is given by the relation (13).

By replacing n by 2 (in the case of two components), we find the model of the literature given by the relation (6).

Since in our application, we have three standby components connected in parallel, by deduction, the relation to be exploited to evaluate the failure rate is given by Equation (14).

$$\lambda(t) = \frac{\lambda^3\cdot t^2}{2+2\lambda t+(\lambda t)^2} \quad (14)$$

Figure 11 shows the variation in the reliability of the system in passive redundancy.

The same observation is made as that made in the previous case. Indeed, the reliability of the system decreases exponentially over time. The optimum reliability for this system is 0.9981, when it is in its early stages of use.

Figure 12 shows the variation in the standby system failure rate.

The failure rate increases dramatically over time.

3.2.3. Comparison of the Two Redundancies

Figure 13 shows the reliability comparison of active redundancy and passive redundancy.

It can be seen that at any time t, the reliability of the system obtained by passive

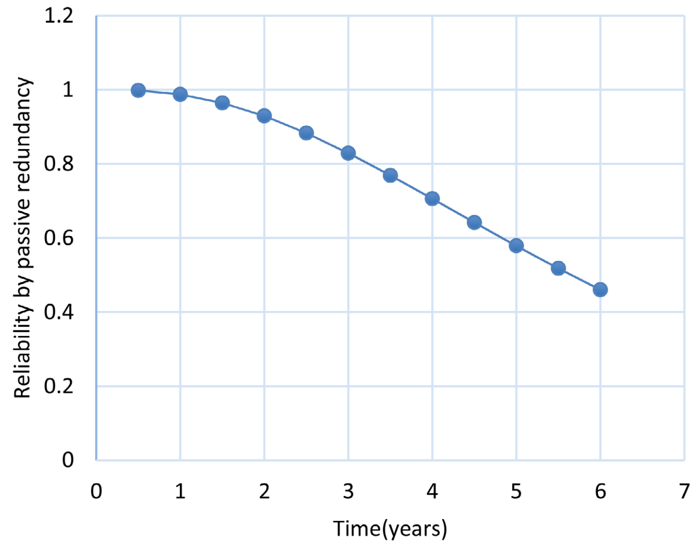

Figure 11. Variation in system reliability by passive redundancy.

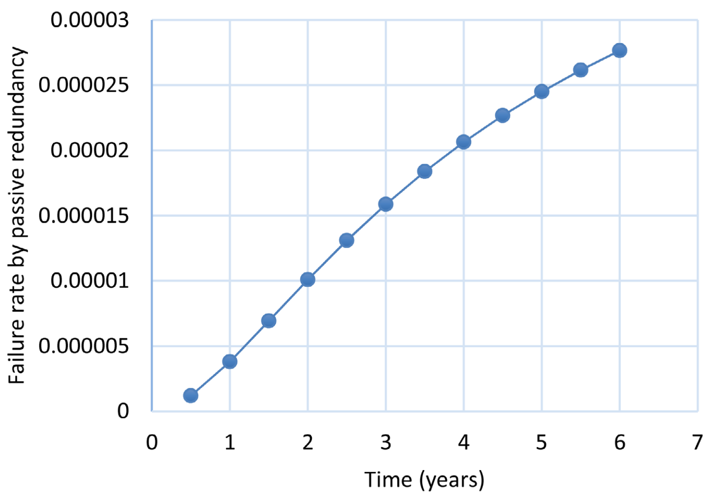

Figure 12. Variation in failure rate by passive redundancy.

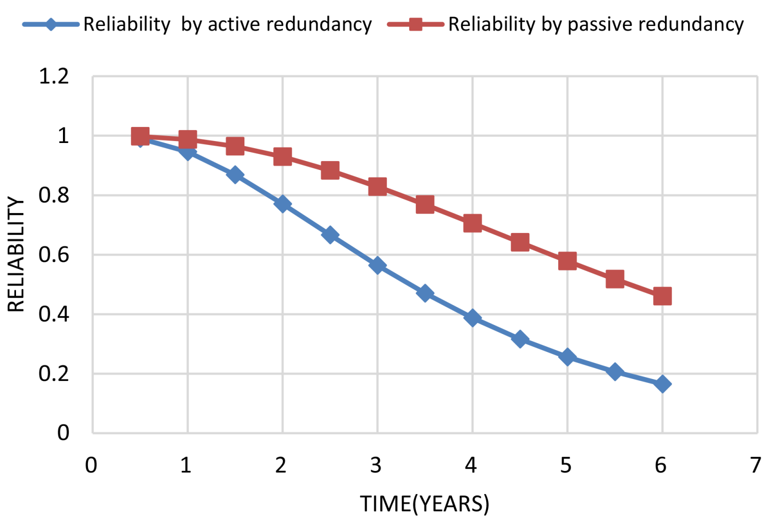

Figure 13. Reliability comparison of active and passive redundancy.

redundancy is much higher than that obtained by active redundancy. We therefore deduce that passive redundancy improves reliability as well as active redundancy.

3.2.4. Gain in Reliability through Redundancies

The need for availability imposes higher levels of reliability implying multiple redundancy. The system reliability law for n components in parallel is expressed by the relation (15).

$$R(t)=1-\prod_{i=1}^{n}\left[1-\mathrm{e}^{-\lambda_i t}\right] \tag{15}$$

If the *n* components arranged in parallel have a constant failure rate, then the equivalent reliability of the system is given by the relation (16).

$$R(t)=1-\left[1-\mathrm{e}^{-\lambda t}\right]^{n} \tag{16}$$

Thus, the average time for the system to operate correctly until failure (*MTTF*) is determined by expression (17).

$$MTTF=\int_{0}^{+\infty}\left[1-\left[1-\mathrm{e}^{-\lambda t}\right]^{n}\right] \tag{17}$$

By changing the variable such as $u=1-\mathrm{e}^{-\lambda t}$,we have:

$$MTTF=\int_{0}^{1}\frac{1-u^{n}}{\lambda(1-u)}\mathrm{d}u=\frac{1}{\lambda}\int_{0}^{1}\sum_{k=0}^{n-1}u^{k}\mathrm{d}u=\frac{1}{\lambda}\sum_{k=0}^{n-1}\int_{0}^{1}u^{k}\mathrm{d}u$$

$$\text{So: } MTTF=\frac{1}{\lambda}\sum_{k=0}^{n-1}\left[\frac{u^{k+1}}{k+1}\right]_{0}^{1}=\frac{1}{\lambda}\sum_{k=0}^{n-1}\frac{1}{k+1}=\frac{1}{\lambda}\left(1+\frac{1}{2}+\cdots+\frac{1}{n}\right) \tag{18}$$

The goal of multiple redundancy is to find the number of components to have in parallel to improve *MTTF*.

If for our case, we admit that we measure reliability at *MTTF*, which is equal to *1/λ*, then by how much will putting in parallel several identical and independent components increase *MTTF*?

For *n = 1*, we find well $MTTF=\frac{1}{\lambda}$. For *n = 2*, we get $MTTF=\frac{1}{\lambda}\left(1+\frac{1}{2}\right)=\frac{3}{2\lambda}$.

So putting 2 components instead of 3 increases the *MTTF* by half. It does not double, contrary to what one might think (it is the passive redundancy of 4 components which makes it possible to double the *MTTF*).

To double the *MTTF*, you need 4 components, because $1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}=2.083$. To multiply the *MTTF* by 10, it would be necessary to put in parallel 123,69 components!

This slow evolution is due to the fact that the series $\sum_{i=1}^{n}\frac{1}{i}$ diverges logarithmically: $\lim_{n\to+\infty}\left(1+\frac{1}{2}+\cdots+\frac{1}{n}-\ln n\right)=0.577215$ (Euler constant).

This means that in practice you cannot increase the operating safety of a system as easily by adding redundancies. In addition, cost constraints must also be taken into account.

4. Conclusion

When we finished writing this paper, we proposed a model for evaluating the failure rate of a standby multi-component system. The application of the first two redundancies has shown that passive redundancy improves reliability better. Regarding the gain in reliability through redundancies, it is the passive redundancy of 4 components which makes it possible to double the *MTTF*. Active redundancy on the other hand of 2 components increases the *MTTF* by half. Furthermore, in practice, it is difficult to increase the operating safety of a system by adding redundancies, because it is necessary to take into account the constraints related to the cost.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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